1. (a) First substitute $u = \sin(x)$ so $du = \cos(x)dx$.

\[ \int \frac{\cos(x)}{\sin^2(x) + 5\sin(x) + 6} \, dx = \int \frac{1}{u^2 + 5u + 6} \, du \]

Next, use a partial fractions expansion of $1/(u^2 + 5u + 6) = 1/(u + 3)(u + 2)$.

\[ \frac{1}{(u + 3)(u + 2)} = \frac{A}{u + 3} + \frac{B}{u + 2} \]

\[ = \frac{A(u + 2)}{(u + 3)(u + 2)} + \frac{B(u + 3)}{(u + 2)(u + 3)} \]

That is,

\[ 1 = A(u + 2) + B(u + 3) \]

Taking $u = -2$ gives $B = 1$; taking $u = -3$ gives $A = -1$. Then

\[ \int \frac{1}{u^2 + 5u + 6} \, du = \int \frac{-1}{u + 3} \, du + \int \frac{1}{u + 2} \, du \]

\[ = -\ln|u + 3| + \ln|u + 2| + C \]

Finally, substituting $u = \sin(x)$ gives

\[ \int \frac{\cos(x)}{\sin^2(x) + 5\sin(x) + 6} \, dx = \left. \frac{-\ln|\sin(x) + 3| + \ln|\sin(x) + 2| + C}{\sin(x)} \right| \]

(b) There is no obvious substitution for $\int \ln(x^2 + 1) \, dx$, so try integration by parts

\[ x = \ln(x^2 + 1) \quad dv = dx \]

\[ dx = \frac{2x}{x^2 + 1} \quad v = x \]

Then

\[ \int \ln(x^2 + 1) \, dx = x \ln(x^2 + 1) - \int \frac{2x^2}{x^2 + 1} \, dx \]

Dividing,

\[ \frac{2x^2}{x^2 + 1} = 2 - \frac{2}{x^2 + 1} \]

Combining these results gives

\[ \int \ln(x^2 + 1) \, dx = x \ln(x^2 + 1) - \int \frac{2x^2}{x^2 + 1} \, dx \]

\[ = x \ln(x^2 + 1) - \int \left( 2 - \frac{2}{x^2 + 1} \right) \, dx \]

\[ = x \ln(x^2 + 1) - 2x + 2 \tan^{-1}(x) \quad \text{by rule 41} \]
2. (a) The $x$-nullcline is $x^2 + y^2 - 4 = 0$, that is, the circle $x^2 + y^2 = 4$. The $y$-nullcline is $y^2 - 1 = 0$, that is $y = \pm 1$.

(b) First, consider the plane divided into regions by the $x$-nullcline. For points inside the circle $x^2 + y^2 = 4$, $x' < 0$ and the vector field points W; for points outside the circle $x' > 0$ and the vector field points E.

Next, consider the plane divided into regions by the $y$-nullcline. If $y > 1$ or $y < -1$, $y' = y^2 - 1 > 0$ and the vector field points N; if $-1 < y < 1$, $y' < 0$ and the vector field points S.

Combining these, we find

3. The trace of the matrix is $\text{tr} = a + 1$ and the determinant is $\text{det} = a + 1$. Consequently, $\text{tr} = \text{det}$. In the trace-determinant plane, this is the straight line of slope 1 passing through the origin.

The parabola $\text{det} = \text{tr}^2/4$ and the line $\text{det} = \text{tr}$ intersect where

$$\text{tr} = \text{tr}^2/4$$

That is,

$$0 = \text{tr}(\text{tr} - 4)$$
\( \det = \frac{t^2}{4} \)

Then the fixed point at the origin is

- a saddle point for \( t < 0 \), that is, for \( a < -1 \),
- an unstable spiral for \( 0 < t < 4 \), that is, for \(-1 < a < 3\), and
- an unstable node for \( 4 \leq t \), that is, for \( 3 \leq a \)

4. Use the box-counting dimension formula:

\[
d(A) = \lim_{n \to \infty} \frac{\log(3^n)}{\log(1/(1/2^n))} = \lim_{n \to \infty} \frac{\log(3^n)}{\log(2^n)} = \frac{\log(3)}{\log(2)}
\]

Then \( d(B) = 2d(A) \) means

\[
\lim_{n \to \infty} \frac{\log(N_B(1/2^n))}{\log(1/(1/2^n))} = 2 \lim_{n \to \infty} \frac{\log(3^n)}{\log(2^n)} = \lim_{n \to \infty} \frac{2 \log(3^n)}{\log(2^n)} = \lim_{n \to \infty} \frac{\log(9^n)}{\log(2^n)}
\]

To achieve this dimension for \( B \), \( N_B(1/2^n) = 9^n \) works.