1. (a) Substitute $u = x + 1$, so $du = dx$ and $x = u - 1$. Then

\[
\int \frac{x}{\sqrt{x+1}}dx = \int \frac{u-1}{\sqrt{u}}du
\]

\[
= \int u^{1/2} - u^{-1/2}du
\]

\[
= \frac{u^{3/2}}{3/2} - \frac{u^{1/2}}{1/2} + C
\]

\[
= \frac{2}{3}(x+1)^{3/2} - 2(x+1)^{1/2} + C
\]

(b) Use integration by parts, with $u = \ln(x)$ and $dv = \sqrt{x}dx$, so $du = dx/x$ and $v = (2/3)x^{3/2}$. Then

\[
\int \sqrt{x}\ln(x)dx = \frac{2}{3}x^{3/2}\ln(x) - \int \frac{2}{3}x^{3/2}\frac{1}{x}dx
\]

\[
= \frac{2}{3}x^{3/2}\ln(x) - \frac{2}{3}\int x^{1/2}dx
\]

\[
= \frac{2}{3}x^{3/2}\ln(x) - \frac{4}{9}x^{3/2} + C
\]

(c) Substitute $u = \sin(x)$ so $du = \cos(x)dx$. Then

\[
\int \frac{\cos(x)}{\sin(x)\sqrt{9 - \sin^2(x)}}dx = \int \frac{du}{u\sqrt{9 - u^2}}
\]

\[
= -\frac{1}{3}\ln \left| \frac{\sqrt{9 - u^2} + 3}{u} \right| + C \quad \text{by rule 30}
\]

\[
= -\frac{1}{3}\ln \left| \frac{\sqrt{9 - \sin^2(x)} + 3}{\sin(x)} \right| + C
\]

2. Because $2 < b + \tau < 3$, the graph of $\phi_{n+1}$ has three branches, and the fixed point occurs where the middle branch intersects the diagonal. The equation of the middle branch is $\phi_{n+1} = b\phi_n + \tau - 1$, so the fixed point is the solution of

\[
\phi_n = b\phi_n + \tau - 1
\]

That is,

\[
\phi_n = \frac{\tau - 1}{1 - b}
\]
Because the slope of every branch of the graph is greater than 1, the fixed point is unstable.

3. For the differential equation

\[
\begin{bmatrix}
x' \\
y'
\end{bmatrix} = \begin{bmatrix}
a & a \\ -1 & 1
\end{bmatrix} \begin{bmatrix}
x \\
y
\end{bmatrix}
\]

the trace and determinant are

\[
tr = a + 1 \quad \text{and} \quad det = 2a
\]

so \(det = 2tr - 2\) for every differential equation in this family. The line \(det = 2tr - 2\) crosses the parabola \(det = tr^2/4\) at

\[
tr = 4 \pm 2\sqrt{2}
\]

From where the line crosses the \(tr\)-axis and the parabola \(det = tr^2/4\), we see the fixed point at the origin is a saddle point for \(tr < 1\), that is, \(a < 0\), an unstable node for \(1 < tr < 4 - 2\sqrt{2}\), that is, \(0 < a \leq 3 - 2\sqrt{2}\), an unstable spiral for \(4 - 2\sqrt{2} < tr < 4 + 2\sqrt{2}\), that is, \(3 - 2\sqrt{2} < tr < 3 + 2\sqrt{2}\), and an unstable node for \(4 + 2\sqrt{2} < tr\), that is, \(3 + 2\sqrt{2} < a\).

4. Suppose \(N_r(A) = N_r(B) + 5\). Because both \(A\) and \(B\) have positive dimensions, \(N_r(A) \to \infty\) and \(N_r(B) \to \infty\) as \(r \to 0\), so \(N_r(B)\) is the larger of \(N_r(B)\) and 5. Then
\[ d(A) = \lim_{r \to 0} \frac{\log(N_r(A))}{\log(1/r)} \]
\[ = \lim_{r \to 0} \frac{\log(N_r(B) + 5)}{\log(1/r)} \]
\[ = \lim_{r \to 0} \frac{\log(N_r(B)(1 + 5/N_r(B)))}{\log(1/r)} \]
\[ = \lim_{r \to 0} \left( \frac{\log(N_r(B))}{\log(1/r)} + \frac{\log(1 + 5/N_r(B))}{\log(1/r)} \right) \]
\[ = d(B) \]

because \( \log(1 + 5/N_r(B)) \leq \log(5) \) and the denominator \( \log(1/r) \) goes to \( \infty \).

5. (a) The system
\[
\begin{align*}
  x' &= x - y^2 \\
y' &= x - y^3
\end{align*}
\]
has \( x \)-nullcline \( x = y^2 \) and \( y \)-nullcline \( x = y^3 \), pictured on the left.

(b) The fixed points are \( (0, 0) \) and \( (1, 1) \).
(c) For \((x, y)\) to the left of the \( x \)-nullcline, \( x < y^2 \), so \( x' < 0 \) and the vector field points left; for \((x, y)\) to the right of the \( x \)-nullcline, \( x > y^2 \), so \( x' > 0 \) and the vector field points right. For \((x, y)\) above the \( y \)-nullcline, \( x < y^3 \), so \( y' < 0 \) and the vector field points down; for \((x, y)\) below the \( y \)-nullcline, \( x > y^3 \), so \( y' > 0 \) and the vector field points up. Combining this information, we have the directions shown on the right.