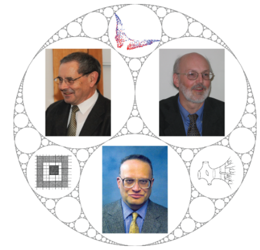

Challenges in Geometry, Analysis and Computation: High Dimensional Synthesis.

Yale University June 4th - 6th, 2012.

Posters

Monday June 4th

Presenter: **Shivkumar Chandrasekaran** (*University of California, Santa Barbara*)

Title: A new technique for the numerical solution of partial differential equations.

Abstract: We present a new technique, based on a clever numerical minimization of certain Sobolev norms, for the numerical solution of partial differential equations. The new method is very high order even for complex geometries and variable coefficient problems. One surprising feature of the method is that we prefer to pose the problem in first-order form. The system does not have to be square as long as the equations are linearly independent and possess a unique solution. We show the efficiency of the method for computing highly accurate solutions by applying it to various classical 2D PDEs including: div-curl, Poisson (both 4x3 and 3x3 formulation), Helmholtz, wave equation (1+1), heat equation (1+1), elastic equation, biharmonic, and linearized Navier-Stokes. What is more amazing about this method is that we use the same code for all of these problems, unlike classical FEM and FD codes that have to be re-written to match the problem type. The entire code is only a few hundred lines of Matlab. We will also solve a div-curl problem with a singular solution that is simultaneously discontinuous on a slit. The problem will be solved with both a single boundary condition and a double boundary condition on the slit. In all cases we will retain high order convergence and show very high accuracy in the solution (including biharmonic problems). The geometries we consider will be very complex, including the exterior region of a car shaped body. We will also show high order convergence on curved geometries.

Presenter: **Yen Do** (*Yale University*)

Title: Quantitative convergence of Fourier series in weighted settings.

Abstract: I will describe some ongoing joint work with Michael Lacey about variational Carleson theorems in weighted spaces.

Presenter: **Adrianna Gillman** (*Darmouth College*)

Title: A fast direct solution technique for two-dimensional quasi-periodic fields.

Abstract: Accurate numerical modeling of periodic scattering problems is important for many applications including solar cells, diffraction gratings, acoustics absorbers, and photonic crystal slabs. Recently, a new integral representation for quasi-periodic scattering problems that achieves spectral accuracy has been developed. This representation avoids the blow-up of the quasi-periodic Greens function at so-called Wood anomalies by restricting the exterior domain to a quasi-periodic strip and using the free-space Greens function. Upon discretization, the integral representation leads to a linear system of size N+M where N is the number of boundary nodes and M is the number of periodizing unknowns. Solving this system can be computationally prohibitive for complicated geometries and near resonances. In this talk, we present a new direct solution technique which utilizes the robust fast direct solvers developed for one-dimensional integral equations. Since M is O(1), the result is a method with computational cost that grows linearly with N. Each additional solve is much less expensive than the first. This is important for problems with multiple right-hand sides that often arise in design problems where there are multiple incident angles. This work is done in collaboration with Alex Barnett.

Presenter: **Sijia Hao** (*University of Colorado at Boulder*)

Title: A high-order Nystrom discretization scheme for boundary integral equations defined on rotationally symmetric surfaces .

Abstract: We propose a scheme for rapidly and accurately computing solutions to boundary integral equations on rotationally symmetric surfaces in $\mathbb{R}^3$. The scheme uses the Fourier transform to reduce the original BIE defined on a surface to a sequence of BIEs defined on a generating curve for the surface. Nystrom discretization is used to discretize the BIEs on the generating curve. The quadrature is a high-order Gaussian rule that is modified near the diagonal to retain high-order accuracy for singular kernels. An accelerated technique for computing the kernels of the reduced BIEs is developed for those kernels arising from Laplace's and Helmholtz equations. The above algorithm can be easily extended to multiply connected domains where each scatter is locally axisymmetric. We utilize high-order Gaussian quadrature to discretize each surface, and then solve the system via an iterative solver based on GMRES but with a block-diagonal preconditioner. The Fast Multipole Method is used to accelerate each iteration. Numerical results are presented for solving exterior Laplace and Helmholtz problems on multiple ellipsoidal and bowl-shaped cavity domains. The poster will describe the method and present numerous numerical examples, including very large multibody scattering simulations.

Presenter: **Jarod Hart** (*University of Kansas, Lawrence*)

Title: Bilinear Littlewood-Paley Estimates and Calderón-Zygmund Theory.

Abstract: This work in harmonic analysis addresses the study of oscillatory behavior of functions in the context of bilinear operators. Bilinear operators are transformations that combine two waves into a new one. Some new almost orthogonality estimates are obtained, which provide understanding of interactions between waves oscillating at different frequencies. Using these estimates we are able to obtain new ways of quantifying properties of the resulting wave in terms of the initial waves. Estimates of this type are a bilinear version of Littlewood-Paley estimates and are used to justify useful frequency decompositions. Among other applications, our Littlewood-Paley estimates give a complete characterization of the continuity of certain operators called bilinear Caldern-Zygmund operators which are of great relevance in harmonic analysis.

Presenter: **Kenneth Ho** (*Courant Institute, NYU*)

Title: A fast direct solver by structured matrix compression.

Abstract: We present a fast direct solver for structured linear systems by multilevel matrix compression. Such matrices are characterized by a hierarchical low-rank block structure and often arise from the discretization of non-oscillatory integral equations. We develop an algorithm that exposes the 'sparsity' of such matrices, relying ultimately on standard sparse direct solver technology for fast inversion. Applications to molecular electrostatics and multiple scattering are demonstrated.

Presenter: **Martin Mohlenkamp** (*Ohio University*)

Title: If the Multiparticle Schrodinger Equation were easy to solve, then Chemistry would be too boring to support life.

Abstract: The multiparticle Schrodinger equation is the basic governing equation in quantum mechanics. Many person-centuries and cpu-millennia have been spent constructing approximate solutions to it. We should be glad it is so hard to solve because its subtle behavior allows the rich Chemistry upon which life depends.

This poster presents the multiparticle Schrodinger equation and explains (some of) the reasons it is difficult to solve: high-dimensionality, antisymmetry, scaling to large systems, inter-particle cusps, singular potentials and nuclear cusps, odd function spaces, etc. We also describe our efforts to overcome these difficulties.

Presenter: **Raanan Schul** (*SUNY, Stony Brook*)

Title: On faking wavelet coefficients for metric space valued functions.

Abstract: For a function f from one Euclidean space to another, studying the coefficients in a wavelet expansion is a very strong tool. We will offer a substitute, suitable for functions which are metric space valued (with Euclidean domain). We will then state (and explain the statements of) several results that use this substitute. Part of this talk is on joint work with Jonas Azzam.

Presenter: **Matthias Taus** (*University of Texas at Austin*)

Title: Isogoemetric Boundary Element Methods on Smooth Domains.

Abstract: Isogeometric analysis has emerged as a framework for integrating computational geometry and finite element methods. In effect, in isogeometric analysis, interpolation functions widely used in computational geometry are adopted as finite element basis functions. At this stage, there are numerous applications of isogeometric analysis to problems of major practical and scientific significance. The premise of this work is that isogeometric analysis is extremely beneficial for boundary element methods, especially for problems defined on smooth boundaries. The main reason is that isogeometric analysis involves smooth basis functions and exact representation of surfaces. We will show that these properties allow one to prove stability of collocation methods on C2 surfaces, represent singular and hypersingular operators in terms of weakly singular ones, construct higherorder approximations without introducing additional degrees of freedom, and formulate integral equations leading to linear algebraic systems whose conditioning is independent of the mesh size. This is joint work with Gregory J. Rodin

Presenter: **Ignacio Uriarte-Tuero** (*Michigan State University*)

Title: Applications of weighted inequalities and corona decompositions in complex and harmonic analysis.

Abstract: I would like to present a few highlights of applications of weighted inequalities and corona decompositions in complex and harmonic analysis. In particular:

1) Astala's conjecture regarding the distortion of Hausdorff measures for planar quasiconformal maps (jointly proved by Lacey, Sawyer and U-T, the proof uses weighted inequalities), and its sharpness (U-T);

2) Sharp metric conditions in terms of Riesz capacities for removability of sets under K-quasiregular maps in the plane (joint works by Tolsa and U-T, and Astala, Clop, Tolsa, U-T, and Verdera, the proofs use weighted inequalities and corona decompositions)

3) A characterization of the two-weighted norm inequality in L^2 for the Hilbert transform in terms of testing over characteristic functions of measurable sets (joint work with Lacey, Sawyer, Shen, the proof uses corona decompositions). It has been conjectured (Nazarov-Treil-Volberg) that it should suffice to test over characteristic functions of intervals.

Presenter: Jianlin Xia (*Purdue University*)

Title: Randomized sparse direct solvers.

Abstract: We propose some new structured direct solvers for large linear systems, using randomization and other techniques. Our work involves new flexible methods to exploit structures in large matrix computations. Our randomized structured techniques provide both higher efficiency and better applicability than some existing structured methods. New efficient ways are proposed to conveniently perform various complex operations which are difficult in standard rank-structured solvers. Applications of the techniques to sparse inversion, least squares problems and eigenvalue problems will also be shown.

We also study the following issues: 1. Develop matrix-free structured solvers. 2. Update a structured factorization when few matrix entries change. 3. Relaxed rank requirements in structured solvers. We show the feasibility of our methods for solving various difficult problems, especially high dimensional ones. 4. Develop effective preconditioners for problems without significant rank structures. We analyze the criterion for compressing off-diagonal blocks so as to achieve nearly optimal effectiveness and efficiency in our preconditioner.

Presenter: Irene Waldspurger (*École Polytechnique, Paris*)

Title: Reconstruction of a function from the modulus of its wavelet transform.

Abstract: Our problem is to reconstruct $f \in L^2(\mathbb{R})$ from the modulus of its wavelet transform, that is, from $Uf = \{|\psi_j \star f|, j \in \mathbb{Z}\}$, where $\psi \in L^2(\mathbb{R})$ and, for all $j \in \mathbb{Z}$, $\psi_j f(x) = \frac{1}{2^j}\psi(\frac{x}{2^j})$. It appears in image processing and a close one has applications in audio processing. We will consider it from a theoretical then practical viewpoint.

We will see that, for a particular choice of ψ, Uf uniquely determines f, up to multiplication by some $\alpha \in \mathbb{C}$ with $|\alpha| = 1$. This relates to Akutowicz's proof [1] of the fact that, on the contrary, a 1-dimensional and compactly supported $f \in L^2(\mathbb{R})$ cannot be recovered from $|\hat{f}|$.

In practice, one can perform reconstruction with Gerchberg and Saxton's algorithm [3], widely used in phase retrieval, which is a succession of orthogonal projections on two sets of $L^2(\mathbb{R})^{\mathbb{N}}$. It always converges but, as the sets are not convex, often returns an incorrect result.

We will present another method, based on convexification techniques and very similar to the one introduced in [2]. It gives far better results but has a complexity of $O(N^{3.5})$ (with N the number of samples) and becomes intractable for N larger than a few hundreds. Therefore, we will describe and discuss a multigrid version, of complexity $O(N \log N)$, which can handle much larger signals.

[1] E. J. Akutowicz. On the determination of the phase of a Fourier integral, I. *Trans. Am. Math. Soc.*, 83 :179-192, 1956.

[2] T. Strohmer, E. J. Candès, Y. C. Eldar and V. Voroninski. Phase retrieval via matrix completion. *Preprint*, 2011.

[3] R. Gerchberg and W. Saxton. A practical algorithm for the determination of phase from image and diffraction plane pictures. *Optik*, 35 :237-246, 1972.

Presenter: **Bo Zhang** (*Duke University*)

Title: Adaptive Scheduling for Parallel Fast Multipole Method.

Abstract: We introduce an adaptive and asynchronous scheduling scheme for the parallelization of the adaptive fast multipole method (FMM). The FMM has enabled computational solutions to increasingly many more application problems at large scales, in diverse areas, since its original introduction by Greengard and Rokhlin 25 years ago. An imperative means to coping with the large scale in problem size, and its growth, is to explore the concurrency in the FMM and exploit parallel computer architectures. In fact, there have been persistent efforts on parallel FMM over the past years as hardware and software architectures evolve continuously or undergo paradigm shifts. The first report of a parallel FMM algorithm (with uniform particle distribution) is by Greengard and Gropp on shared-memory architectures in 1990. Among the most recent reports is a petascale direct numerical simulation of blood flow on 200K cores and heterogeneous architectures by the research team led by Biros. Following the Greengard-Gropp scheme for parallel scheduling of the FMM, a common feature in the subsequent works is that the operations at one level of the spatial partition hierarchy (the tree) are separated from those at the next coarser or finer level by an explicitly or implicitly placed barrier. There are O(logN) such barriers in a parallel calculation of N-particle interactions. Previously existing parallel scheduling schemes differ mainly in load balancing, especially for the adaptive FMM. The workload is characterized in various ways, such as by the particle population, or the nodes of the partition tree (namely, the nonempty boxes), or the edges of the tree. These parallel scheduling schemes are predetermined, tightly coupled with the evolved, and still evolving, computing systems, and grow increasingly complex. The effectiveness and efficiency, however, are diminishing, even on a shared-memory system, as the number of cores increases with the advance in computer engineering and architectures.

In the parallel scheduling scheme introduced here, the adaptation takes place at two stages. At the parallelization stage, the scheme is adaptive to the basic features of a particular algorithm as well as the basic architectural characteristics of a target computing system. In the execution process, the scheme is adaptive to the dynamic or random variations in numerical data, processing components, or other sources with some degree of uncertainty that cannot be predicted or predetermined precisely and efficiently. Such adaptabilities are achieved by a simplified model with universality. The model has two key components, one for each of the main stages we are concerned with. The first component renders an absolute ranking among algorithm components without architectural constraints, and a conditioned ranking subject to the constraint in the number of available processing units. The second component provides relative ranking and dynamic prioritization. Pertinent to the FMM, we use spatio-temporal graphs to depict and help explain the scheduling analysis and schemes.

When applied to the FMM, the new scheme breaks down the levelwise barriers in operations at the parallelization stage and surpass the performance barriers at the execution. We present experimental results of the parallel FMM performance on a shared memory system in a multithreaded processing environment. The demonstrated particle simulations assume the screened Coulomb interactions with particles randomly distributed in two prototype geometric domains, a cube and a partial sphere surface. The parallel performance is evaluated in weak and strong scaling measures.

This is joint work with Jingfang Huang, Nikos P. Pitsianis and Xiaobai Sun.

Tuesday June 5th

Presenter: **Dominque Duncan** (*Yale University*)

Title: **Predicting Seizures in Intracranial EEG Data Using Diffusion Maps.**

Abstract: Often medication or surgery are not viable options for patients with epilepsy, thus it is important to find a reliable tool to predict seizures. This way the patient may be warned at least a few minutes prior to the seizure and take the necessary precautions. Finding an accurate predictor of seizures has become a major focus of research during the last few decades. The goal of this study is to predict a seizure from intracranial EEG (icEEG) data. A novel approach is proposed that capitalizes on the diffusion map framework, which was recently presented and is considered to be one of the leading manifold learning methods to-date. Diffusion mapping provides dimensionality reduction of the data as well as pattern recognition that may be used to distinguish different states of the patient, for example, resting and preseizure. Based on diffusion maps, a new nonlinear independent component analysis (ICA) algorithm is developed, which enables to construct coordinates that generate efficient geometric representations of the complex structures underlying in the icEEG data. In addition, this method is adapted to the noisy icEEG data and enables to extract the brain activity. The algorithm is tested on icEEG data recorded from several electrodes from patients being evaluated for possible epilepsy surgery at the Yale-New Haven Hospital. Artifacts are removed from the data prior to the analysis. Preliminary results show that the proposed approach provides a distinction between resting and preseizure states.

Presenter: **Armin Eftekhari** (*Colorado School of Mines*)

Title: **The Restricted Isometry Property for Block Diagonal Matrices.**

Abstract: In compressive sensing (CS), the restricted isometry property (RIP) is a condition on measurement operators which ensures that robust recovery of sparse vectors is possible from noisy, undersampled measurements via computationally tractable algorithms. It is by now well-known that Gaussian random matrices satisfy the RIP under certain conditions on the number of measurements. Their use is limited in practice, however, due to storage limitations, computational considerations, or the mismatch of such matrices with certain measurement architectures. These issues have recently motivated considerable efforts towards studying RIP for structured random matrices. In this paper, we study the RIP for block diagonal measurement matrices where each block on the main diagonal is itself a sub-Gaussian random matrix. Our main result states that such matrices can indeed satisfy the RIP but that the requisite number of measurements depends on certain properties

of the basis in which the signals are sparse. In the best case, these matrices perform nearly as well as dense Gaussian random matrices despite having many fewer nonzero entries.

Presenter: **Dimitris Giannakis** (*Courant Institute, NYU*)

Title: **Capturing intermittent and low-frequency variability in high-dimensional data through nonlinear Laplacian spectral analysis.**

Abstract: Nonlinear Laplacian spectral analysis (NLSA) is a method for spatiotemporal analysis of high-dimensional data, which represents spatial and temporal patterns through singular value decomposition of a family of maps acting on scalar functions on the nonlinear data manifold. Through the use of orthogonal basis functions (determined by means of graph Laplace-Beltrami eigenfunction algorithms) and time-lagged embedding, NLSA captures intermittency, rare events, and other nonlinear dynamical features which are not accessible through classical linear approaches such as singular spectrum analysis. We present applications of NLSA to detection of decadal and intermittent variability in the North Pacific sector of comprehensive climate models, and dimension reduction of a chaotic low-order model of the atmosphere. This is joint work with Andrew Majda.

Presenter: **Roy Lederman** (*Yale University*)

Title: **A random-projections-like approach to problems in the reading of the DNA.**

Abstract: In recent years, sequencing, the procedure of reading the DNA has become very widely used in biology. Typically, sequencing machine procedures produce a large number ($> 10^6$) of short DNA strings (about 100 characters-long). Read alignment, the process of matching these short strings to a known reference genome, is often considered to be a time consuming and computationally intensive part of sequencing. Over the years, many software packages for fast alignment have been developed. However, as the throughputs of sequencing keep increasing, alignment is still a bottleneck. We present a randomized approach to nearest-neighbors-searches in libraries of strings, and the application of this approach to the alignment problem. While most of the existing algorithms and software packages for read alignment are based on prefix-trees and hash tables, our algorithm uses random permutations of strings. We discuss some of the conceptual similarities and distinctions between using random permutations for searches in libraries of strings and using random projections for searches in $\mathbb{R}^n$. We also present a prototype implementation, designed for the alignment problem in biology. Experiments on real data and simulated data, suggest that this implementation is faster and often more accurate than existing software packages.

Presenter: **Yitzchak Lockerman** (*Yale University*)

Title: **Case Study: Real Time Texture Extraction Using Real Time Diffusion Manifolds.**

Abstract: We demonstrate that geometric diffusion based calculations can be performed in real time on a large scale. As a case study, we use them for the purpose of texture extraction, the process of identifying samples of a single texture within an image. By utilizing Graphical Processing Units (GPUs) and an appropriate problem formulation, our implementation is able to produce results over 300 times as fast as past published results. We also examine the benefits of using GPUs over traditional CPUs. Our implementation can serve as a model for other applications where geometric diffusion calculations are required.

Presenter: **Mauro Maggioni** (*Duke University*)

Title: Multiscale Geometric Methods for high dimensional data.

Abstract: Data sets are often modeled as samples from a probability distribution in $\mathbb{R}^D$, for D large. It is often assumed that the data has some interesting low-dimensional structure, for example that of a d-dimensional manifold $\mathcal{M}$, with d much smaller than D. When $\mathcal{M}$ is simply a linear subspace, one may exploit this assumption for encoding efficiently the data by projecting onto a dictionary of d vectors in $\mathbb{R}^D$ (for example found by SVD), at a cost $(n + D)d$ for n data points. When $\mathcal{M}$ is nonlinear, there are no "explicit" and algorithmically efficient constructions of dictionaries that achieve a similar efficiency: typically one uses either random dictionaries, or dictionaries obtained by black-box global optimization. In this paper we construct data-dependent multi-scale dictionaries that aim at efficiently encoding and manipulating the data. Their construction is fast, and so are the algorithms that map data points to dictionary coefficients and vice versa, in contrast with L^1-type sparsity-seeking algorithms, but alike adaptive nonlinear approximation in classical multiscale analysis. In addition, data points are guaranteed to have a compressible representation in terms of the dictionary, depending on the assumptions on the geometry of the underlying probability distribution. This poster is associated to a paper accepted in ACHA Sep. 2011. This is joint work with W. K. Allard and G. Chen.

Presenter: **Michael McCoy** (*Caltech*)

Title: Sharp recovery bounds for convex deconvolution, with applications.

Abstract: Consider the common situation where observed data consists of the superposition of two signals. Some examples include: an image of the night sky containing both stars and galaxies; a communications message with impulsive noise; and a low rank matrix obscured by sparse corruptions. Deconvolution is the problem of determining the constituent signals from their superposition. A fundamental question is "When is deconvolution possible with a tractable algorithm?"

We describe a convex optimization framework for deconvolution, and provide a geometric characterization of success in this framework. This geometric viewpoint coupled with a random incoherence model yields an exact formula for the probability that our method succeeds. This formula leads to simple summary statistics that reveal sharp phase transitions between success and failure.

We apply our results to deconvolving the superposition of sparse vectors in random bases, a stylized robust communications protocol, and determining a low rank matrix corrupted by a matrix that is sparse in a random basis. We show that empirical results closely match our theoretical bounds. This is joint work with Joel A. Tropp.

Presenter: **Deanna Needell** (*Claremont McKenna College*)

Title: Robust image recovery via total variation minimization.

Abstract: Compressed sensing is a new field which shows that reliable, nonadaptive data acquisition, with far fewer measurements than traditionally assumed, is possible. In this talk we will introduce the fundamental ideas behind compressed sensing, focusing on imaging techniques, as well as new results for imaging via totalvariation. Discrete images, composed of patches of slowly-varying pixel values, have sparse or compressible wavelet representations which allow the techniques from compressed sensing such as L1-minimization to be utilized. In addition, such images also have

sparse or compressible discrete derivatives which motivate the use oftotal variation minimization for image reconstruction. Although image compression is a primary motivation for compressed sensing, stability results for total-variationminimization do not follow directly from the standard theory. In this talk, we present provable near-optimal reconstruction guarantees for total-variation minimization using properties of the bivariate Haar transform along with numerical studies demonstrating its advantages.

Presenter: **Aviv Rotbart** (*Tel Aviv University*)

Title: Hierarchical data organization, clustering and denoising via Coarse-grained localized diffusion.

Abstract: Data-analysis methods nowadays are expected to deal with increasingly large amounts of data. Such massive datasets often contain many redundancies. One effect from these redundancies is the high dimensionality of datasets, which is handled by dimensionality reduction techniques. Another effect is the duplicity of very similar observations (or data-points) that can be analyzed together as a cluster. We propose an approach for dealing with both effects by coarse-graining the popular Diffusion Maps (DM) dimensionality reduction framework from the data-point level to the cluster level. This way, the size of the analyzed dataset is decreased by only referring to clusters instead of individual data-points. Then, the dimensionality of the dataset can be decreased by the DM embedding. We show that the essential properties (e.g., ergodicity) of the underlying diffusion process of DM are preserved by the coarse-graining. We propose an implementation for methodology, called Localized Diffusion Folders (LDF). This methodology, whose localized folders are called diffusion folders (DF), introduces consistency criteria for hierarchical folder organization, and clustering and classification of high-dimensional datasets. The DF are multi-level data partitioning into local neighborhoods that are generated by several random selections of data points and DF in a diffusion graph and by redefining local diffusion distances between them. This multi-level partitioning defines an improved localized geometry for the data and a localized Markov transition matrix that is used for the next time step in the advancement of the hierarchical diffusion process. The result of this clustering method is a bottom-up hierarchical data organization where each level in the hierarchy contains LDF of DF from the lower levels. This methodology preserves the local neighborhood of each point while eliminating noisy spurious connections between points and areas in the data affinities graph. One of our goals in this paper is to illustrate the impact of the initial affinities selection on data graphs definition and on the robustness of the hierarchical data organization. This process is similar to filter banks selection for signals denoising. The performance of the algorithms is demonstrated on real data and it is compared to existing methods. The proposed solution is generic since it fits a large number of related problems where the source datasets contain high-dimensional data. This is joint work with Guy Wolf, Gil David and Amir Averbuch and based on the papers: [1] Gil David, Amir Averbuch, Hierarchical data organization, clustering and denoising via localized diffusion folders, Applied and Computational Harmonic Analysis, Volume 33, Issue 1, July 2012, Pages 1-23. [2] Guy Wolf, Aviv Rotbart, Gil David, Amir Averbuch, Coarse-grained localized diffusion, Applied and Computational Harmonic Analysis, Available online 1 March 2012.

Presenter: **Yi Grace Wang** (*University of Minnesota*)

Title: Robust Hybrid Linear Modeling with Applications to Images.

Abstract: We study the related problems of denoising images corrupted by impulsive noise and blind inpainting (i.e., inpainting when the deteriorated region is unknown). Our basic approach is to model the set of patches of pixels in an image as a union of low-dimensional subspaces, corrupted by sparse but perhaps large magnitude noise. For this purpose, we develop a robust and iterative method for single subspace modeling and extend it to an iterative algorithm for modeling multiple subspaces. We prove convergence for both algorithms and carefully compare our methods with other recent ideas for such robust modeling. We demonstrate state of the art performance of our method for both imaging problems.

Presenter: **Han Lun Yap** (*Georgia Tech*)

Title: **The Restricted Isometry Property for Echo State Networks with Applications to Sequence Memory Capacity.**

Abstract: The ability of networked systems (including artificial or biological neuronal networks) to perform complex data processing tasks relies in part on their ability to encode signals from the recent past in the current network state. Here we use Compressed Sensing tools to study the ability of a particular network architecture (Echo State Networks) to stably store long input sequences. In particular, we show that such networks satisfy the Restricted Isometry Property when the input sequences are compressible in certain bases and when the number of nodes scale linearly with the sparsity of the input sequence and logarithmically with its dimension. Thus, the memory capacity of these networks depends on the input sequence statistics, and can (sometimes greatly) exceed the number of nodes in the network. Furthermore, input sequences can be robustly recovered from the instantaneous network state using a tractable optimization program (also implementable in a network architecture).

Presenter: **Teng Zhang** (*University of Minnesota*)

Title: **Robust subspace recovery by optimization on a geodesically convex function.**

Abstract: We introduce an M-estimator to robustly recover the underlying linear model from a data set contaminated by outliers. We prove that the objective function of this estimator is geodesically convex on the manifold of all positive definite matrices, and propose a fast algorithm that obtains its unique minimum. Besides, we prove that when inliers (i.e., points that are not outliers) are sampled from a subspace and the percentage of outliers is bounded by some number, then under some very weak assumptions this algorithm can recover the underlying subspace exactly. We also show that our algorithm compares favorably with other convex algorithms of robust PCA empirically.

Presenter: **Rachel Ward** (*University of Texas at Austin*)

Title: **Reliable image recovery from undersampled measurements.**

Abstract: Compressed sensing provides theoretical guarantees for image reconstruction from undersampled linear measurements. These guarantees depend on the underlying image sparsity (e.g., in the discrete gradient), which is not necessarily known a priori. We discuss how certain statistical procedures like cross validation can be naturally incorporated into compressed sensing to verify the reconstruction error directly and estimate underlying model parameters. This procedure is justified by the "near equivalence" of two central concepts in compressed sensing: restricted isometries and

Johnson-Lindenstrauss embeddings. The near-equivalence has additional implications that should be of independent interest.

Wednesday June 6th

Presenter: **Afonso Bandeira** (*Princeton University*)

Title: **Cheeger Inequality for the Graph Connection Laplacian and its Applications to Stable Phaseless Reconstruction.**

Abstract: The $O(d)$ Synchronization problem consists of estimating a set of unknown orthogonal transformations O_i from noisy measurements of a subset of the pairwise ratios $O_iO_j^{-1}$. We formulate and prove a Cheeger-type inequality that relates a measure of how well it is possible to solve the $O(d)$ synchronization problem with the spectra of an operator, the graph Connection Laplacian. We also show how this inequality provides a worst case performance guarantee for a spectral method to solve this problem. This is joint work with Amit Singer (Princeton) and Daniel Spielman (Yale) . These guarantees also play a major role on showing the stability of a certain method to solve the problem of reconstruction without phase The latter part is joint work with: Boris Alexeev (Princeton), Dustin Mixon (Princeton) and Matthew Fickus (Air Force Inst. Tech.).

Presenter: **Xiuyuan Cheng** (*Princeton University*)

Title: **The Spectrum of Random Inner-product Kernel Matrices.**

Abstract: We consider $n \times n$ matrices whose (i, j)-th entry is $f(X_i^T X_j)$, where $X_1, ..., X_n$ are i.i.d. standard Gaussian random vectors in $\mathbb{R}^p$, and f is a real-valued function. The eigenvalue distribution of these random kernel matrices is studied at the "large p, large n" regime. It is shown that, when p and n go to infinity, $p/n = \gamma$ which is a constant, and f is properly scaled so that $Var(f(X_i^T X_j))$ is $O(p^{-1})$, the spectral density converges weakly to a limiting density on $\mathbb{R}$. The limiting density is dictated by a cubic equation involving its Stieltjes transform. While for smooth kernel functions the limiting spectral density has been previously shown to be the Marcenko-Pastur distribution, our analysis is applicable to non-smooth kernel functions, resulting in a new family of limiting densities.

Presenter: **Gilad Lerman** (*University of Minnesota*)

Title: **Robust Computation of Linear Models.**

Abstract: Consider a dataset of vector-valued observations that consists of a modest number of noisy inliers, which are explained well by a low-dimensional subspace, along with a large number of outliers, which have no linear structure. We describe a convex optimization problem that can reliably fit a low-dimensional model to this type of data. When the inliers are contained in a low-dimensional subspace we provide a rigorous theory that describes when this optimization can recover the subspace exactly. We present an efficient algorithm for solving this optimization problem, whose computational cost is comparable to subspace recovery by PCA. We also show that the sample complexity of the proposed subspace recovery is of the same order as PCA subspace recovery and we consequently obtain some nontrivial robustness to noise. This presentation is based on

three joint works: 1) with Teng Zhang, 2) with Michael McCoy, Joel Tropp and Teng Zhang, and 3) with Matthew Coudron.

Presenter: **Mauro Maggioni** (*Duke University*)

Title: **Multiscale Geometric and Spectral Analysis of Plane Arrangements.**

Abstract: Modeling data by multiple low-dimensional planes is an important problem in many applications such as computer vision and pattern recognition. In the most general setting where only coordinates of the data are given, the problem asks to determine the optimal model parameters, estimate the model planes, and cluster the data accordingly. Though many algorithms have been proposed, most of them need to assume prior knowledge of the model pa- rameters and thus address only the last two components of the problem. In this paper we propose an accurate and efficient algorithm based on multi- scale SVD analysis and spectral methods to tackle the problem." This poster is associated to a paper accepted in CVPR Jul. 2011. This is joint work with G. Chen.

Presenter: **Alain Plattner** (*Princeton University*)

Title: **Spatiospectral concentration of vector fields on a sphere.**

Abstract: We pose and solve the analogue of Slepian's time-frequency concentration problem for vector fields on the surface of the unit sphere, to determine an orthogonal family of strictly bandlimited vector fields that are optimally concentrated within a closed region of the sphere or, alternatively, of strictly spacelimited functions that are optimally concentrated in the vector spherical harmonic domain. Such a basis of simultaneously spatially and spectrally concentrated functions should be a useful data analysis and representation tool in a variety of geophysical and planetary applications, as well as in medical imaging, computer science, cosmology, and numerical analysis. The vector spherical Slepian functions can be found by solving either an algebraic eigenvalue problem in the spectral domain or a Fredholm integral equation in the spatial domain. The associated eigenvalues are a measure of the spatiospectral concentration. For general regions the tangential vector solutions are decoupled from the radial ones, which are identical to the spherical scalar Slepian functions. When the concentration region is an axisymmetric polar cap, the eigenvalue problem can be constructed analytically and the associated matrix simplifies into a block-diagonal form. This enables the eigenfunctions to be computed accurately and efficiently.

Presenter: **Neta Rabin** (*Yale University*)

Title: **Synthesizing high dimensional data and functions from embedding coordinates with adaptive Laplacian pyramids. .**

Abstract: We consider the problem of synthesizing high dimensional data points from a low-dimensional embedding representation. The coordinates in low dimensional space provide a reliable space for analyzing and simulating the data. However, in some application there is a need to find the reverse statistics and to synthese functions and data in the latent space. In this work, we use multi-scale Gaussian kernels, which are adapted to the geometry of the low-dimensional domain, to approximate the high dimensional data, which is treated as a vector function on the embedding coordinates. An example for synthesizing day ahead 24-hour electricity price profiles demonstrates the proposed method. This is joint work with Ronald R. Coifman.

Presenter: **Andrei Tarfulea** (*Princeton University*)

Title: **Conservation Equations through Adaptive Grid Points.**

Abstract: Conservation laws play important roles in modeling physical systems. In principle, they say that the increase of a physical quantity is equal to its change in flux. In many applications (e.g., traffic flow, fluid dynamics, etc.) the flux depends on the physical quantity, and this leads to differential equations. Often these equations develop regions where the measured quantity changes rapidly over small distances; such regions are called viscous shocks. A small amount of viscosity in the model ensures the solution will be smooth (and unique), though its gradient may grow arbitrarily large. From a numerical standpoint, this poses many challenges. In computing the solution, we make heavy use of a Fourier Continuation Method, which is stable, fast, and high-order accurate provided the solution maintains a degree of smoothness with respect to the mesh. To accommodate solutions with shocks, we created a smoothly adapting mesh. Over time, the sample points in the mesh migrate to the regions where the solution varies rapidly (i.e., where it develops shocks). We present a numerical solver using an effective mesh adaptation in one dimension. We also explore an alternate approach in 2D that easily generalizes to arbitrary dimensions yet does not exceed the computational requirements of the standard Fourier techniques.

Presenter: **Lanhui Wang** (*Princeton University*)

Title: **A Fourier-based Approach for Iterative 3D Reconstruction from Cryo-EM Images.**

Abstract: A major challenge in single particle reconstruction methods using cryo-electron microscopy is to attain a resolution sufficient to interpret fine details in three-dimensional (3D) macromolecular structures. Obtaining high resolution 3D reconstructions is difficult due to unknown orientations and positions of the imaged particles, possible incomplete coverage of the viewing directions, high level of noise in the projection images, and limiting effects of the contrast transfer function of the electron microscope. In this paper, we focus on the 3D reconstruction problem from projection images assuming an existing estimate for their orientations and positions. We propose a fast and accurate Fourier-based Iterative Reconstruction Method (FIRM) that exploits the Toeplitz structure of the operator $\mathbf{A}^*\mathbf{A}$, where $\mathbf{A}$ is the forward projector and $\mathbf{A}^*$ is the back projector. The operator $\mathbf{A}^*\mathbf{A}$ is equivalent to a convolution with a kernel. The kernel is pre-computed using the non-uniform Fast Fourier Transform and is efficiently applied in each iteration step. The iterations by FIRM are therefore considerably faster than those of traditional iterative algebraic approaches, while maintaining the same accuracy even when the viewing directions are unevenly distributed. The time complexity of FIRM is comparable to the direct Fourier inversion method. Moreover, FIRM combines images from different defocus groups simultaneously and can handle a wide range of regularization terms. We provide experimental results on simulated data that demonstrate the speed and accuracy of FIRM in comparison with current methods. This is joint work with Yoel Shkolnisky and Amit Singer.

Presenter: **Hau-tieng Wu** (*Princeton University*)

Title: **Vector Diffusion Maps, Connection Laplacian and their applications.**

Abstract: We introduce vector diffusion maps (VDM), a new mathematical framework for organizing and analyzing massive high dimensional data sets, images and shapes. VDM is a mathematical and algorithmic generalization of diffusion maps and other non-linear dimensionality reduction methods, such as LLE, ISOMAP and Laplacian eigenmaps. While existing methods are either directly or indirectly related to the heat kernel for functions over the data, VDM is based on the heat kernel for 1-forms and vector fields. VDM provides tools for organizing complex data sets, embedding them in a low dimensional space, and interpolating and regressing vector fields over the data. In particular, it equips the data with a metric, which we refer to as the *vector diffusion distance*. In the manifold learning setup, where the data set is distributed on (or near) a low dimensional manifold $\mathcal{M}^d$ embedded in $\mathbb{R}^p$, we prove the relation between VDM and the connection-Laplacian operator for 1-forms over the manifold. The algorithm is directly applied to the cryo-EM problem and the result will be demonstrated. This is a joint work with Amit Singer.

Presenter: **Ying Xiao** (*Georgia Tech*)

Title: Structure from Local Optima: Factoring Distributions and Learning Subspace Juntas.

Abstract: Independent Component Analysis (ICA), a well-known approach in statistics, assumes that data is generated by applying an affine transformation of a fully independent set of random variables, and aims to recover the orthogonal basis corresponding to the independent random variables. We consider a generalization of ICA, wherein the data is generated as an affine transformation applied to a product of distributions on two orthogonal subspaces, and the goal is to recover the two component subspaces. Our main result, extending the work of Frieze, Jerrum and Kannan, is an algorithm for generalized ICA that uses local optima of high moments and recovers the component subspaces. When one component is on a k-dimensional relevant subspace and satisfies some mild assumptions while the other is noise modeled as an $(n-k)$- dimensional Gaussian, the complexity of the algorithm is $T(k, \epsilon) + poly(n)$ where T depends only on the k-dimensional distribution. We apply this result to learning a k-subspace junta, i.e., an unknown 0-1 function in $\mathbb{R}^n$ determined by an unknown k-dimensional subspace. This is a common generalization of learning a k-junta in $\mathbb{R}^n$ and of learning an intersection of k halfspaces in $\mathbb{R}^n$, two important problems in learning theory. Our main tools are the use of local optima to recover global structure, a gradient-based algorithm for optimization over tensors, and an approximate polynomial identity test. Together, they significantly extend ICA and the class of k-dimensional labeling functions that can be learned efficiently. This is joint work with Santosh Vempala.

Presenter: **Xu Yang** (*Courant Institute, NYU*)

Title: A large deviation framework to analyze metastable/excursion behavior in climate system.

Abstract: We studied the dynamic transition/excursion phenomena in climate systems. We built a framework using large deviation theory, in which different climate regimes are represented by the statistical most likely states and the transition is described by the most likelihood pathways connecting either metastable states or target sets in the small noise limit. Specifically we considered an energy-constrained stochastic dynamics (equilibrium statistical system), the most likely states of whose invariant measure coincide with the selective decay states. We compute the transition pathways using a constrained String method. Nonequilibrium statistical climate systems were also analyzed where the transition pathways were computed by the geometric minimum action method.

Presenter: **Yuan Yao** (*Peking University*)

Title: L1-norm Statistical Ranking vs. Angular Embedding on Random Graphs.

Abstract: Finding a global rating/ranking, as one-dimensional embedding of data, based on pairwise comparisons is a fundamental problem in computer vision, decision science, game theory, social choice theory, financial economics, machine learning, psychology, and statistics. Least square (or L2) ranking receives rising attention due to its simplicity but subtle structure, such as Hodge decomposition on graphs. Angular Embedding maps pairwise comparison data onto the circle and find the global ranking score via a primary eigenvector solution, which is a kind of complex reciprocal matrix approach whose real version was used in Economics and decision science. Both work under the presence of small Gaussian noise. However recently Yu (2011) shows that Angular Embedding (AE) is much more robust than least square ranking against sparse outliers. In fact, for complete graphs Singer (2011) established a fact that AE is optimal in approximately recovering the signals against sparse but uniform noise on circles. Although in robust statistics Least Absolute Deviation (L1) has been exploited since 1970s to remove sparse outliers, the L1-formulation in Yu (2011) is incorrect for such a purpose and thus inferior to AE in experiments. Therefore, it is unclear what's the robustness property of L1-norm ranking compared with AE in the same setting.

In this report we present a comparative study on L1-ranking and AE in the setting of sparse outliers occurred on Erdös-Rényi random graphs, as an extension from Singer's setting. We propose a new L1-ranking formulation which solves the Least Absolute Deviation of gradient flows on graphs from pairwise comparison flows which has been explored by Hochbaum and Osher et al. recently. Our results show that the proposed L^1-ranking undergoes a phase-transition from exact recovery to failure when the sparsity drops. With some upper and lower bounds such a phase-transition is shown to be optimal up to a logarithmic factor. From random matrix theory, AE is shown to achieve approximate recovery with the same phase-transition rate. Experimental studies further show better performance of L1-ranking than AE in exact recovery, against sparse outliers and even with additional small Gaussian noise. These results show that L1-norm statistical ranking is a good candidate for robust ranking. This is joint work with Jiechao Xiong and Xiuyuan Cheng.

Presenter: **Jane Zhizhen Zhao** (*Princeton University*)

Title: Rotationally Invariant Image Representation for Viewing Angle Classification in Cryo-Electron Micrsoscopy.

Abstract: We introduce a new rotationally invariant viewing angle classification method for identifying, among a large number of noisy Cryo-EM projection images, similar views without prior knowledge of the molecule. Each image is first expanded in an orthonormal steerable basis to generate expansion coefficients. Rotating an image is equivalent to phase shifting the expansion coefficients. Thus we are able to extend the theory of bispectrum of 1D periodic signal to 2D image. The randomized PCA algorithm efficiently reduces the dimensionality of the bispectrum coefficients and enables fast computation of the similarity between any pair of images. The initial nearest neighbors are found by using an approximate nearest neighbor search algorithm. The rotational alignment is only performed for images with their nearest neighbors. The initial nearest neighbor classification and alignment are further improved by a new classification method called vector diffusion map (VDM) that takes into account the consistency of the group transformations.