## Math 116 Midterm Exam

1. Evaluate these integrals. If you use the integral table, give the number of the integral used.

(a)

$$\int \frac{x}{x^2 - 4x - 5} dx$$
$$\int \sin(x) \cos(x) e^{\sin(x)} dx$$

(c)

(b)

$$\int \frac{dx}{\cos(x)\sin(x)\sqrt{1-\tan^2(x)}}$$

2. Consider this differential equation

$$x' = xy$$
$$y' = x^2 - y^2$$

(a) Write the equations for the nullclines and sketch the nullclines. In your sketch indicate which is the x-nullcline and which is the y-nullcline.

(b) The nullclines divide the plane into several regions. For each region say whether the vectors point NE, NW, SE, or SW. Give reasons to support your answers.

3. For the differential equation

$$\begin{bmatrix} x'\\y' \end{bmatrix} = \begin{bmatrix} a^2/4 & 1\\ -a & 0 \end{bmatrix} \begin{bmatrix} x\\y \end{bmatrix}$$

list the range of a values for which the origin is an asymptotically stable node, an asymptotically stable spiral, a center, an unstable spiral, an unstable node, and a saddle point.

4. (a) Suppose the number of boxes of side length  $1/2^n$  needed to cover a shape is  $N(1/2^n) = 2^n + 3^n$ . Compute the box-counting dimension of this shape.

(b) Find a number K so that if the number of boxes of side length  $1/2^n$  needed to cover another object is  $N(1/2^n) = 2^n + 3^n + K^n$ , then this object will have dimension twice that of the object of part (a) of this problem.