Convergence Practice Answers & Hints

1. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2+1}}$ converges by comparison to $\sum \frac{1}{n^2}$

2. $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2-1}}$ converges by limit comparison to $\sum \frac{1}{n^2}$

3. $\sum_{n=1}^{\infty} \frac{6^n}{n^n}$ converges by the root test

4. $\sum_{n=1}^{\infty} \frac{n!}{6^n}$ diverges by the ratio test

5. $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ converges by the integral test, $u = \ln x$

6. $\sum_{n=2}^{\infty} \frac{\ln n}{n^3}$ converges by comparison to $\sum \frac{1}{n^2}$, noting $\frac{\ln n}{n} < 1$

7. $\sum_{n=1}^{\infty} \frac{(n+3)!}{n!\,3^n}$ converges by the ratio test

8. $\sum_{n=2}^{\infty} \frac{n}{(\ln n)^n}$ converges by the root test, using $n^{1/n} \to 1$

9. $\sum_{n=1}^{\infty} \frac{(n!)^n}{(n^n)^2}$ diverges by the root test