Math 116 Practice Midterm 1

1. Evaluate these integrals. If you use the integral table, give the number of the integral used.

(a)

$$\int \frac{x+1}{x^2+3x-10}dx$$

(b)

$$\int e^{2x}\cos(e^x)dx$$

(c)

$$\int \frac{x}{\sqrt{1+x^4}}dx$$

2. For the Arnold cardiac model

$$\phi_{n+1} = f(\phi_n) = b\phi_n + \tau (\text{mod } 1)$$

with $2 < b + \tau < 3$, suppose ϕ_0 and ϕ_1 are a 2-cycle, with ϕ_0 in the left interval and ϕ_1 in the right interval. Find ϕ_0 in terms of b and τ.

3. For the differential equation

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1+a & 1 \\ a & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

determine the range of a values for which the origin is an unstable node, an asymptotically stable node, an unstable spiral, an asymptotically stable spiral, a center, and a saddlepoint. Note that not every type of fixed point may occur in this problem.

4. Consider the differential equation

$$\frac{dx}{dt} = x^3 - kx^2$$

where k is a positive constant.

(a) Find the x-values of the fixed points of the system.

(b) Determine if each fixed point is stable, unstable, or neither.

5. Consider the system

$$x' = (y-x)(x^2+y^2-1)$$
$$y' = y + x$$

(a) Find the nullclines and sketch them. Indicate which are the x-nullclines and which are the y-nullclines.

(b) On your sketch, the nullclines divide the plane into several regions. In each region, indicate whether the vectors point NE, NW, SW, or SE.