Math 116 Practice Midterm 2

1. Evaluate these integrals. If you use the integral table, give the number of the integral used.

(a)
$$\int \frac{x}{\sqrt{x+1}} dx$$

(b)
$$\int \sqrt{x} \ln(x) dx$$

(c)
$$\int \frac{\cos(x)}{\sin(x)\sqrt{9-\sin^2(x)}} dx$$

2. For the Arnold cardiac model

$$\phi_{n+1} = f(\phi_n) = b\phi_n + \tau \pmod{1}$$

with $2 < b + \tau < 3$ and $0 < \tau < 1$, find the ϕ value of the fixed point in terms of b and τ , and determine the stability of this fixed point.

3. For the differential equation

$$\begin{bmatrix} x'\\y' \end{bmatrix} = \begin{bmatrix} a & a\\-1 & 1 \end{bmatrix} \begin{bmatrix} x\\y \end{bmatrix}$$

determine the range of *a* values for which the origin is an unstable node, an asymptotically stable node, an unstable spiral, an asymptotically stable spiral, a center, and a saddlepoint. Note that not every type of fixed point may occur in this problem.

4. Suppose $N_r(A)$ and $N_r(B)$ denote the smallest number of boxes of side length r needed to cover shapes A and B. If $N_r(A) = N_r(B) + 5$, how are the dimensions of A and B related? Suppose both A and B have dimensions greater than 0. Support your answer.

5. Consider the system

$$x' = x - y^2$$
$$y' = x - y^3$$

(a) Find the nullclines and sketch them. Indicate which are the *x*-nullclines and which are the *y*-nullclines.

(b) On your sketch for (a), indicate the fixed points. Find the coordinates of the fixed points.

(c) On your sketch, the nullclines divide the plane into several regions. In each region, indicate whether the vectors point NE, NW, SW, or SE.