Math 116 Practice Midterm 4

1. Evaluate these integrals. If you use the integral table, give the number of the integral used.

(a)

$$\int \frac{x}{x^2 + 4x - 5} dx$$
1. (b)

$$\int e^{\tan(x)} \sec^2(x) dx$$
1. (c)

$$\int e^{\tan(x)} \frac{\sin(x)}{\cos^3(x)} dx$$

2. Suppose A is the product of a Cantor middle-thirds set along the x-axis and the interval [0, 1] along the y-axis, and B is the product of a Canrot middle-halves set along the x-axis and the interval [0, 2] along the y-axis. Which has the larger box-counting dimension, A or B? Give a reason for your answer.

3. Pictured here is the graph of $f(x) = r(x^2 - x^3)$ for some value of r for which $x_{n+1} = f(x_n)$ has three fixed points.



(a) Find the x values of these fixed points as functions of r.

(b) Without computing derivatives or doing any messy algebra, show the middle fixed point always is unstable. Hint: look at the graph. What do you see?

4. Consider the system

$$\frac{dx}{dt} = x^3 - xy^2$$
$$\frac{dy}{dt} = yx^2 - y$$

(a) (10pts) Find the nullclines and sketch them. Indicate which are the x-nullclines and which are the y-nullclines.

(b) Locate the fixed points of the system. Find their coordinates.

5. Consider the system

$$\begin{bmatrix} x'\\y'\end{bmatrix} = \begin{bmatrix} 1 & a\\b & 1\end{bmatrix} \begin{bmatrix} x\\y\end{bmatrix}$$

where a and b are constants. In the graph below, shade the regions for which the origin is an unstable spiral for this system. Explain how you arrived at your answer.

