Math 116 Practice Final 1

1. Evaluate these integrals. If you use the integral table, give the number of the integral used.

(a)

$$\int \sec^2(t)\cos(\tan(t))dt$$

(b)

$$\int t^3\sin(t^2)dt$$

(c)

$$\int \frac{\sqrt{t^4-1}}{t^2}2tdt$$

2. For the differential equation

$$dx/dt = y - 1 + x^2$$
$$dy/dt = xy$$

(a) Sketch the nullclines, indicating which is the x-nullcline and which is the y-nullcline.

(b) Find the fixed points and determine their stability.

3. Suppose a population is divided into three states, A, B, and C, and that transitions between states have these probabilities.

$$\begin{array}{ccc} Pr(A\to A)=0.8 & Pr(B\to A)=0.2 & Pr(C\to A)=0 \\ Pr(A\to B)=0.2 & Pr(B\to B)=0.5 & Pr(C\to B)=0.1 \\ Pr(A\to C)=0 & Pr(B\to C)=0.3 & Pr(C\to C)=0.9 \end{array}$$

Find the eventual distribution of the population among these three states.

4. The origin is the only fixed point for this differential equation.

$$x' = x + y - (x^2/4) - x(x^2+y^2)$$
$$y' = -x + y - y(x^2+y^2)$$

Find a trapping region that is an annulus, that is, the region between two concentric circles. Deduce this differential equation has a limit cycle.

5. Consider the differential equation $x' = x - t + t^2$ with $x(0) = 1$.

(a) Find the power series expansion for $x(t)$.

(b) Find an expression for $x(t)$ as a sum of exponential and polynomials.

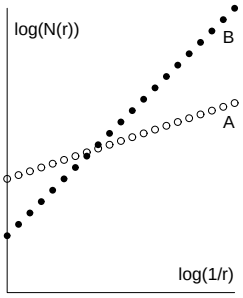
6. (a) Does this series converge or diverge? Why?

$$\sum_{n=2}^{\infty}\frac{1}{\sqrt{n^3-n^2}}$$

(b) Find the sum of this series.

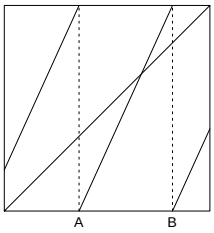
$$1 + 2 \cdot \frac{1}{3} + 3 \cdot \frac{1}{3^2} + 4 \cdot \frac{1}{3^3} + 5 \cdot \frac{1}{3^4} + \cdots$$

7. (a) This graph shows the log-log plots for computing the box-counting dimensions of two fractals. Which has the higher dimension, and why?



(b) Suppose $N_r(A)$ and $N_r(B)$ denote the number of boxes of side length r needed to cover the fractals A and B. These are not the A and B of part (a). Find a relation between $N_r(A)$ and $N_r(B)$ for which the dimensions satisfy $d(A) = 2d(B)$.

8. This is a graph of an Arnold model with $2 < b + \tau < 3$.



(a) Suppose $\{\phi_1, \phi_2\}$ is a 2-cycle, with $\phi_1 < A$ and $\phi_2 > B$. Find the values of ϕ_1 and ϕ_2 in terms of b and τ.

(b) Find the coordinates of the points A and B.