Math 116 Practice Final 2

1. Evaluate these integrals. If you use the integral table, give the number of the integral used.

(a)

$$\int \frac{x+1}{2x^2-3x-2}dx$$

(b)

$$\int \sin(x)\cos(x)\ln(\sin(x))dx$$

(c)

$$\int \frac{1}{x\ln(x)\sqrt{1+(\ln(x))^2}}dx$$

2. Find the radius and interval of convergence of

$$\sum_{n=1}^{\infty} \frac{n3^n}{n^2+1}(2x-1)^n$$

3. For the differential equation

$$dx/dt = y-1$$
$$dy/dt = y-x^2$$

(a) Sketch the nullclines, indicating which is the x-nullcline and which is the y-nullcline.

(b) Find the fixed points and determine their stability.

4. Show the system

$$x' = (3/2)x + y - 2x(x^2+y^2)$$
$$y' = -x + y - 2y(x^2+y^2)$$

has a limit cycle. You may assume the origin is the only fixed point.

5. Assuming the system starts in state 1, find the eventual distribution of the states in the system determined by this Markov process.

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1/3 & 1/2 & 0 & 1/4 & 0 \\ 2/3 & 0 & 1/3 & 0 & 2/3 \\ 0 & 1/2 & 0 & 3/4 & 0 \\ 0 & 0 & 2/3 & 0 & 1/3 \end{bmatrix}$$

6. Find a power series solution $x(t) = \sum_{n=0}^{\infty} a_n t^n$ for the differential equation

$$x' = 2x - t$$

subject to the condition $x(0) = 1$. Express your solution as a sum of a polynomial and an exponential.

7. Determine the stability of the fixed point at the origin

$$x' = -x^3 - 4y - x^5$$
$$y' = -y^3 + 2x - y^5$$

8. For this version of the SIS model

$$S' = S + rI - mSI$$
$$I' = I - rI + mSI$$

where r is the per capita recovery rate, and m is the probability that an encounter between an infecteed and a susceptible results in the infection of the susceptible.

(a) Find the non-zero fixed point.

(b) Determine the stability and type of this fixed point as a function of r and m.