Math 116 Practice Final 3

1. Evaluate these integrals. If you use the table of integrals, give the number of the rule you use.

1. (a)

$$\int \cos(x)\sin(x)e^{\sin(x)}dx$$

(b)

$$\int \frac{\sec^2(x)}{\sqrt{4-\tan^2(x)}}dx$$

(c)

$$\int \frac{e^x}{e^{2x}-3e^x+2}dx$$

2. Find the radius and interval of convergence of

$$\sum_{n=1}^{\infty} \frac{1}{n3^n}(5x-1)^n$$

3. For the system

$$dx/dt = y - x^3$$
$$dy/dt = y - x$$

(a) Sketch the nullclines, indicating which is the x-nullcline and which is the y-nullcline.

(b) Locate the fixed points on the nullcline plot. Find the coordinates of the fixed points.

(c) Determine the stability of these fixed points.

4. (a) Draw the transition graph of the Markov process with this transition matrix.

$$\begin{bmatrix} .1 & 0 & 0 & 0 \\ .9 & .1 & 0 & 0 \\ 0 & .5 & .1 & .8 \\ 0 & .4 & .9 & .2 \end{bmatrix}$$

(b) Find the eventual distribution of the states in the system determined by this Markov process. Express your answers as fractions, not decimals.

5. Show the system

$$x' = x + 5y - x(x^2+y^2)$$
$$y' = -x + y - y(x^2+y^2)$$

has a limit cycle. You may assume that the origin is the only fixed point.

6. Find a power series solution for the differential equation

$$x'(t) = x(t) + 2t - t^2, \qquad x(0) = 1$$

Express your solution as the sum of an exponential and a polynomial.

7. Consider the differential equation

$$x' = -x^3 + 2xy^2$$
$$y' = 2x^2y - 6y^3$$

(a) Compute the eigenvalues of the derivative matrix at the fixed point $(0, 0)$.

(b) By using a simple Liapunov function, show the origin is asymptotically stable.

8. Consider a population consisting of uninfected cells U, infected cells I, and virus particles V. Uninfected cells are produced at a constant rate, virus particles infect uninfected cells, infected cells produce virus particles, and all uninfected cells, infected cells, and virus particles die. Suppose

a = constant rate at which uninfected cells are produced
b = per capita death rate of uninfected cells
c = rate at which virus particles infect uninfected cells
d = per capita death rate of infected cells
e = per capita rate at which infected cells produce virus particles
f = per capita death rate of virus particles

Write differential equations for U', I', and V'. Justify each term in your equations. Hint: don't forget the principle of mass action.