Math 116 Practice Final 4

1. Evaluate these integrals

(a) $\displaystyle\int \frac{x+1}{x^3-3x^2+2x}dx.$

(b) $\displaystyle\int x^2\cos(x)dx$

(c) $\displaystyle\int \frac{\cos(x)}{\sin(x)\sqrt{1+\sin^2(x)}}dx$

2. Suppose 95% of all colorectal cancer patients under age 60 have a mutated adenomatous polyposis coli (APC) gene, and 50% of people under age 60 not having colorectal cancer have a mutated APC gene. Suppose 10% of people under 60 have colorectal cancer. If a randomly selected person under 60 has a mutated APC gene, what is the probability that person has colorectal cancer?

3. Find the radius and interval of convergence of

$$\sum_{n=1}^{\infty}\frac{2^n}{n^2+4}(x-2)^n$$

4. For the system

$$dx/dt = y - x^3$$
$$dy/dt = x - y^2$$

(a) Sketch the nullclines, indicating which is the x-nullcline and which is the y-nullcline.

(b) Locate the fixed points on the nullcline plot. Find the coordinates of the fixed points.

(c) Determine the stability of the fixed points.

5. Find the eventual distribution of the states in the system determined by the Markov process with this transition matrix

$$\begin{bmatrix} .5 & .2 & .8 & 0 \\ .5 & .8 & .1 & .9 \\ 0 & 0 & .1 & 0 \\ 0 & 0 & 0 & .1 \end{bmatrix}$$

Hint: first draw the transition graph for this system.

6. Show the system

$$x' = 2x - 2y - x(x^2+y^2)$$
$$y' = x + 2y - y(x^2+y^2)$$

has a limit cycle. You may assume that the origin is the only fixed point.

7. Find a power series solution $x(t) = \sum_{n=0}^{\infty} a_n t^n$ for the differential equation

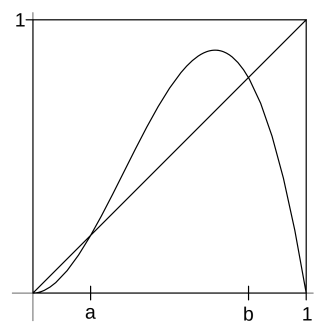
$$x'(t) = tx(t) + t, \qquad x(0) = 1$$

The expressions for the a_n should include factorials.

8. Pictured here is the graph of the population equation

$$P_{n+1} = rP_n^2(1 - P_n)$$

for $r = 6$. Note the system has three fixed points, $P = 0$, $P = a$, and $P = b$.



(a) Describe the long-term behavior of the population for any $P_0 < a$. Give a reason to support your answer.

(b) By drawing an appropriate graphical iteration plot, show there are P_0 near $P = 1$ having the same long-term behavior as that of the P_0 in part (a).