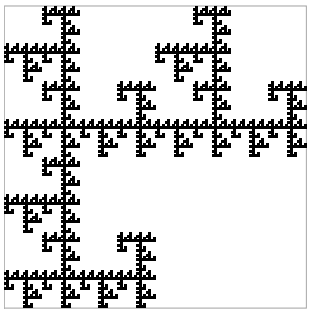
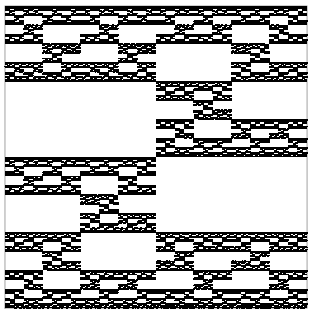
Math 190 Midterm

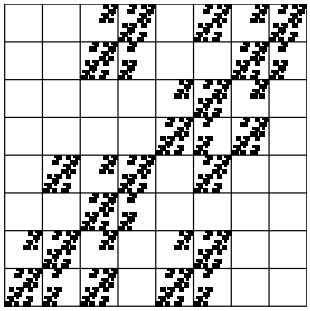
1. Find the IFS rules to generate these fractals. For reference, the unit square is drawn in grey around each fractal.

(a) 

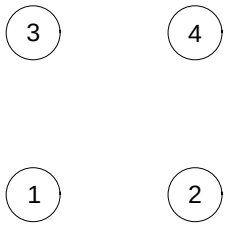
(b) 

2. Find the similarity dimensions of the fractals (a) and (b) of Problem 1. If the Moran equation is used, solve it exactly using the quadratic formula. Leave your answers in terms of logarithms; don't give decimal expressions.

3. (a) Pictured below is an IFS with memory images. Show this image can be generated by forbidden pairs. Explain how you arrived at your answer. Give explicit details. For reference, the length three address squares are shown on both images.



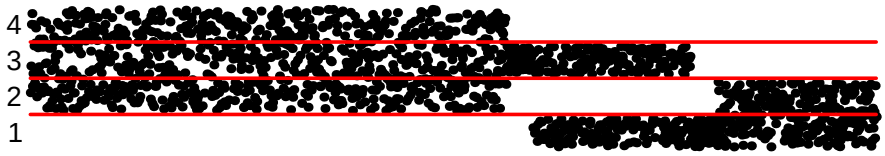
(b) Fill in the appropriate arrows on the transition graph.



(c) Based on properties of this transition graph, show this fractal can be generated by an IFS without memory. Give explicit details.

(d) Give the table for the IFS of part (c).

4. Pictured here is a time series, with the bin boundaries indicated. Below that, in the square with length 2 address regions indicated, sketch the driven IFS this time series would produce. Explain which features of the driven IFS come from which features of the time series. Provide enough detail in the driven IFS sketch to illustrate your explanation.



5. Suppose A and B are Cantor sets each consisting of $N = 2$ pieces scaled by a factor of r. Find the exact value of r for which

$$\dim(A \times B) = \frac{2}{3}$$

Your answer may contain logarithms. Show the calculations by which you arrived at your answer.

6. Sketch the $f(\alpha)$ curve for this IFS with probabilities. Find expressions for $\alpha_{\min}$, $\alpha_{\max}$, $f(\alpha_{\min})$, $f(\alpha_{\max})$, and the maximum value of $f(\alpha)$. Do not give decimal expressions of the answers. For example, write $\log(3)/\log(2)$ instead of 1.58496. Numerical hint: $\log(.05)/\log(.25) > \log(.45)/\log(.5)$.

r	s	θ	φ	e	f	prob
0.5	0.5	0	0	0	0	0.45
0.5	0.5	0	0	0.5	0	0.45
0.25	0.25	0	0	0	0.75	0.05
0.25	0.25	0	0	0.75	0.75	0.05