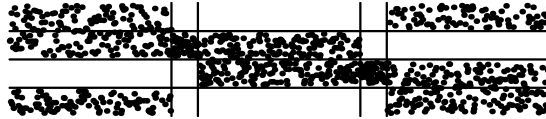
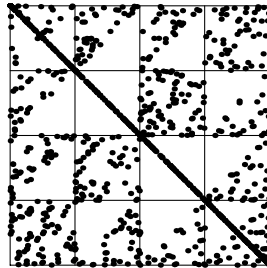


Third homework set solutions

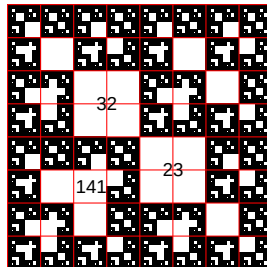
1. Evidently, the time series exhibits five regimes. In the first regime, points are scattered randomly among bins 1, 3, and 4. In the second, all points lie in bin 3. In the third, points are scattered randomly among bins 2 and 3. In the fourth, all points lie in bin 2. In the fifth regime, points are scattered randomly among bins 1, 2, and 4.



Time series points in the first regime produce driven IFS points on the gasket with corners 1, 3, and 4. That is, the corner points are $(0, 0)$, $(0, 1)$, and $(1, 1)$. The second regime produces driven IFS points converging to corner 3, that is, point $(0, 1)$. The third regime produces driven IFS points on the line between corners 3 and 2, that is, between points $(0, 1)$ and $(1, 0)$. The fourth regime produces driven IFS points converging to corner 2, that is, point $(1, 0)$. The fifth regime produces driven IFS points on the gasket with corners 1, 2, and 4, that is, points $(0, 0)$, $(1, 0)$, and $(1, 1)$.

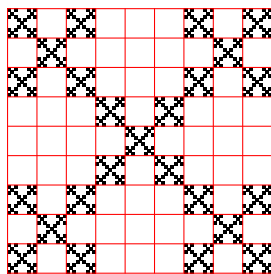


2. The forbidden pairs are 23 and 32. We see that the triple 141 is empty. This triple contains neither 23 nor 32, so this IFS cannot be generated by 1-step memory.



3. We count 5 boxes of side length $1/3$, 25 boxes of side length $1/9$, and 125 boxes of side length $1/27$. The general pattern is

$$N(1/3^n) = 5^n$$



Then the box-counting dimension is

$$\begin{aligned} d_b &= \lim_{n \rightarrow \infty} \frac{\log(N(1/3^n))}{\log(1/(1/3^n))} \\ &= \lim_{n \rightarrow \infty} \frac{\log(5^n)}{\log(3^n)} \\ &= \lim_{n \rightarrow \infty} \frac{n \log(5)}{n \log(3)} \\ &= \frac{\log(5)}{\log(3)} \end{aligned}$$