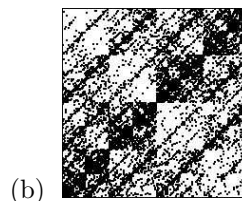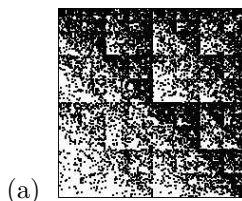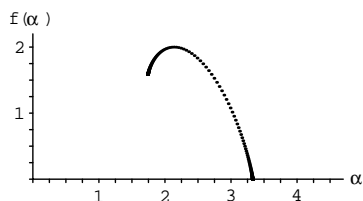Fifth homework set

Due at the beginning of class on Thursday, Oct 11. No late homework will be accepted.

Fold your homework paper vertically and PRINT your name on the outside.

1. Does this $f(\alpha)$ curve correspond to multifractal (a) or (b) below? Explain how you arrived at your answer.

(a)

(b)

2. Consider the multifractal generated by this IFS.

r	s	θ	φ	e	f	prob
1/3	1/3	0	0	0	0	3/20
1/3	1/3	0	0	1/3	0	3/20
1/3	1/3	0	0	2/3	0	3/20
1/3	1/3	0	0	0	1/3	3/20
1/3	1/3	0	0	1/3	1/3	3/20
1/3	1/3	0	0	2/3	1/3	3/20
1/3	1/3	0	0	0	2/3	1/30
1/3	1/3	0	0	1/3	2/3	1/30
1/3	1/3	0	0	2/3	2/3	1/30

(a) Find the maximum value of $f(\alpha)$. Explain how you arrived at your answer.

(b) Find $f(\alpha_{\max})$. Explain how you arrived at your answer.

(c) Find $f(\alpha_{\min})$. Explain how you arrived at your answer.

3. Suppose we construct a randomized Cantor this way. Start with the unit interval and at each stage, replace every interval by two subintervals. The left is scaled by r_1, where $r_1 = 1/2$ with probability $1/2$ and $r_1 = 1/4$ with probability $1/2$. Independently of the choice of r_1, the right is scaled by r_2, where $r_2 = 1/2$ with probability $1/4$ and $r_2 = 1/4$ with probability $3/4$.

(a) Find the maximum and minimum possible values for the dimension of a randomized Cantor set constructed this way.

(b) What is the probability of obtaining either the maximum or minimum dimension by this randomized Cantor set construction?

(c) Find the expected value of the dimension of a randomized Cantor set constructed this way.