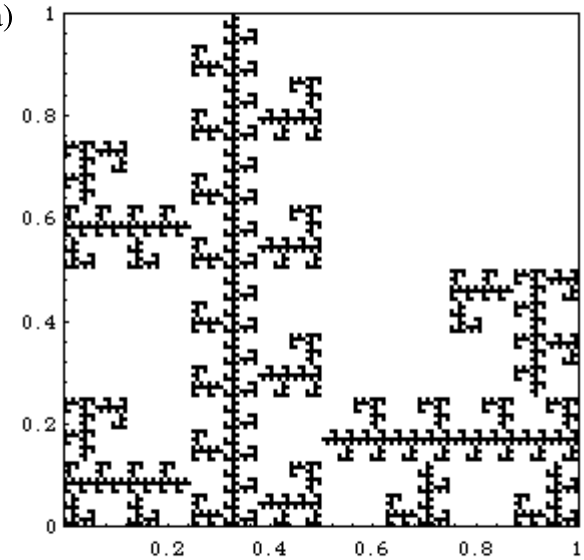
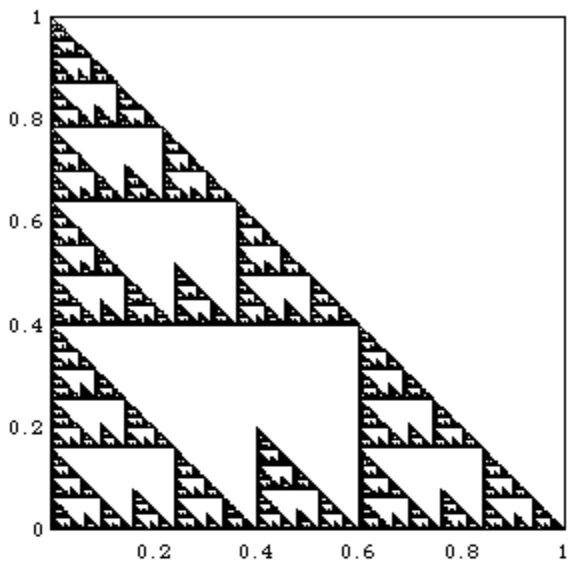
Practice Exam 1 Solutions

1. (a)



R	S	Theta	Phi	E	F
-0.5	0.5	0	0	.5	0
0.5	0.5	90	90	1	0
-0.5	0.5	0	0	0.5	0.5

(b)



R	S	Theta	Phi	E	F
0.6	0.6	0	0	0	0.4
0.4	0.4	0	0	0	0
0.4	0.4	0	0	0.6	0
0.2	0.2	0	0	0.4	0

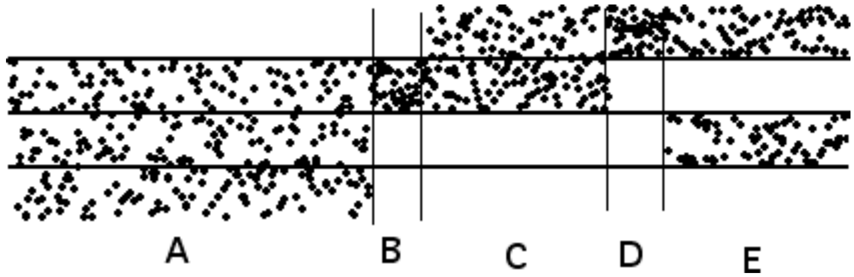
2. (a) There are three pieces, each scaled by 0.5. The dimension is given by the similarity dimension formula d = Log(N)/Log(1/r) = Log(3)/Log(2).

(b) $r_1 = 0.6$, $r_2 = r_3 = 0.4$, $r_4 = 0.2$. Because some of the r_i are different, we must use the Moran equation

$$.6^d + 2*.4^d + .2^d = 1.$$

These scaling factors are not integer powers of a common number. Proving this is a bit tricky, so simply stating it suffices.

3. The time series is divided into five regimes.



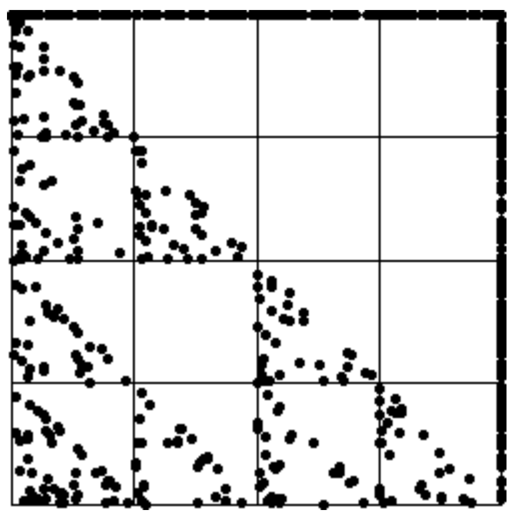
For points in A, all combinations of T_1, T_2, and T_3 are applied, giving a gasket with corners (0,0), (1,0), and (0,1).

For points in B, only T_3 is applied and the driven IFS points move toward corner (0,1).

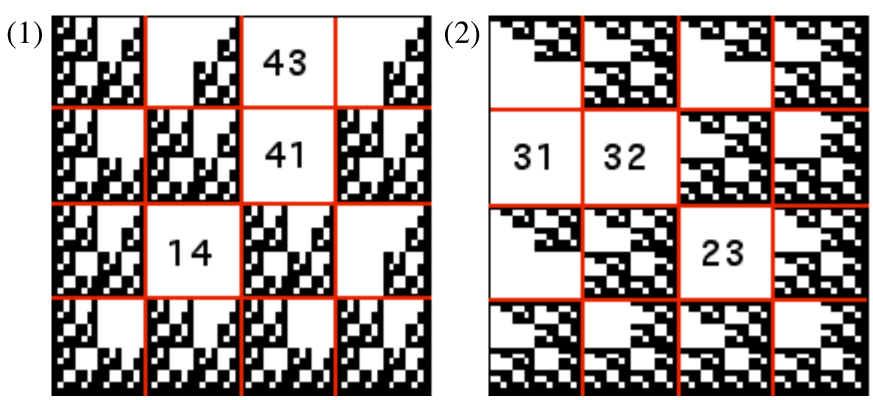
For points in C, all combinations of T_3 and T_4 are applied, giving the line between (0,1) and (1,1).

For points in D, only T_4 is applied and the driven IFS points move toward corner (1,1).

For points in E, all combinations of T_2 and T_4 are applied, giving the line between (1,0) and (1,1).



4. From the driven IFS

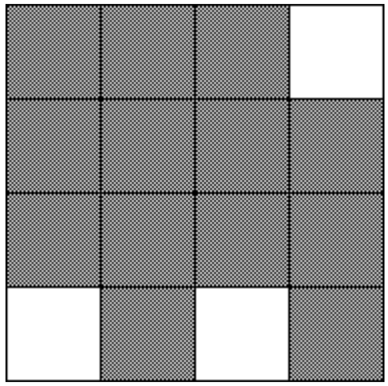


we see (1) has empty length 2 addresses 14, 41, and 43, while (2) has empty length 2 addresses 23, 31, and 32.

So in (1) these combinations are forbidden:

T_1 cannot follow T_4, T_4 cannot follow T_1, and T_4 cannot follow T_3.

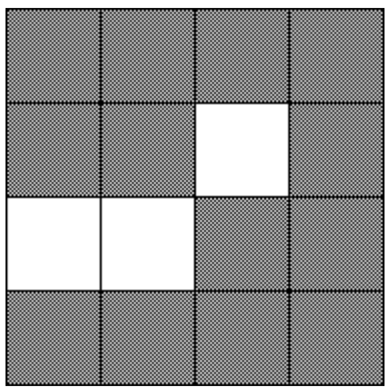
This is indicated by table (i)



In (2) these combinations are forbidden:

T_2 cannot follow T_3, T_3 cannot follow T_1, and T_3 cannot follow T_2.

This is indicated by table (ii)



5. (a) Apply twice the intersection rule:

$$\dim(A \cap (B \cap C)) = \dim(A) + \dim(B \cap C) - n = \dim(A) + (\dim(B) + \dim(C) - n) - n = \dim(A) + \dim(B) + \dim(C) - 2n$$

(b) $\dim(A \cap B \cap C) = 3\ \mathrm{Log}(3)/\mathrm{Log}(2) - 2 \cdot 2 = 0.755$.

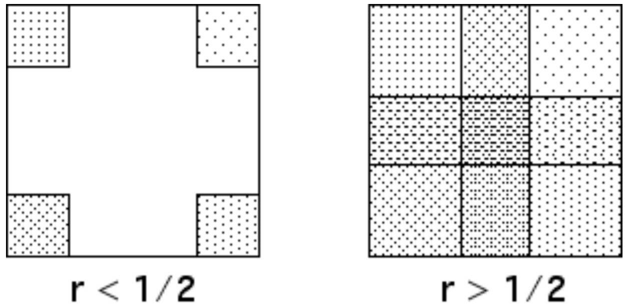
(c) $\dim(A \cap B \cap C) = 3\ \mathrm{Log}(3)/\mathrm{Log}(2) - 2 \cdot 3 = -1.245$.

6. For $N = 4$ the similarity dimension formula gives $d = \mathrm{Log}(4)/\mathrm{Log}(1/r)$.

If $r > 1/2$, then $1/r < 2$ so $\mathrm{Log}(1/r) < \mathrm{Log}(2)$ and we have

$$d = \mathrm{Log}(4)/\mathrm{Log}(1/r) > \mathrm{Log}(4)/\mathrm{Log}(2) = \mathrm{Log}(2^2)/\mathrm{Log}(2) = 2\mathrm{Log}(2)/\mathrm{Log}(2) = 2$$

If $r > 1/2$, we see the pieces of the fractal overlap:



The similarity dimension formula works only if the pieces do not overlap too much. For r > 1/2 the overlap is too large.