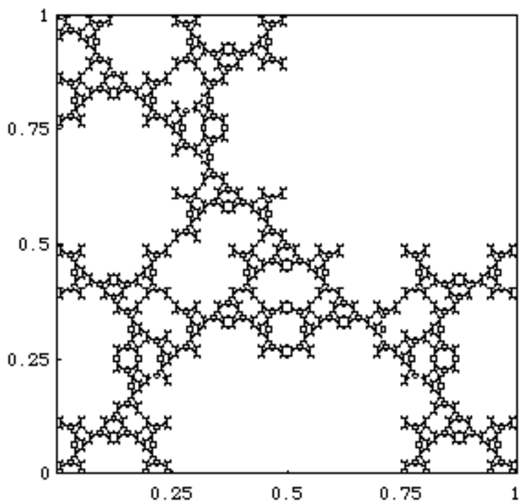
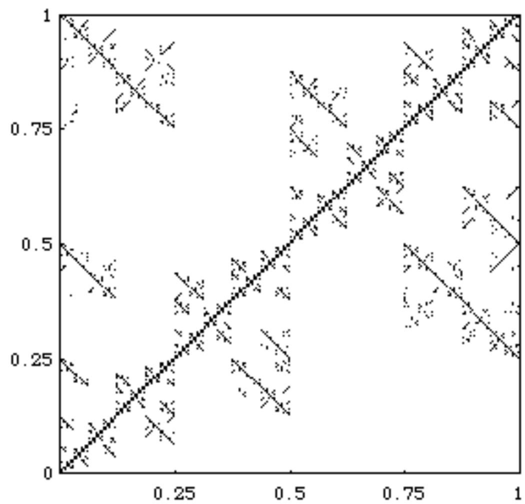
Practice Exam 2 Solutions

1. Here are IFS rules to generate each of these fractals.



R	S	Theta	Phi	E	F
0.5	0.5	-90	-90	0.0	0.5
-0.5	0.5	90	90	1.0	0.5
-0.5	0.5	90	90	0.5	1.0



R	S	Theta	Phi	E	F
0.5	0.5	0	0	0.0	0.0
0.5	0.5	180	180	1.0	1.0
0.25	-0.25	0	0	0.0	1.0
-0.25	0.25	0	0	1.0	0.25

2. (a) The first fractal consists of N=3 pieces each scaled by a factor of r = 1/2. Consequently, the similarity dimension is d = Log(3)/Log(2).

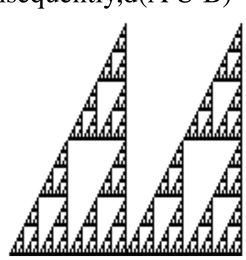
The second fractal consists of 2 pieces scaled by 1/2 and 2 pieces scaled by 1/4. The similarity dimension, d, is given by the Moran equation $2*(.5^d) + 2*(.25^d) = 1$.

(b) The box-counting dimension of A is

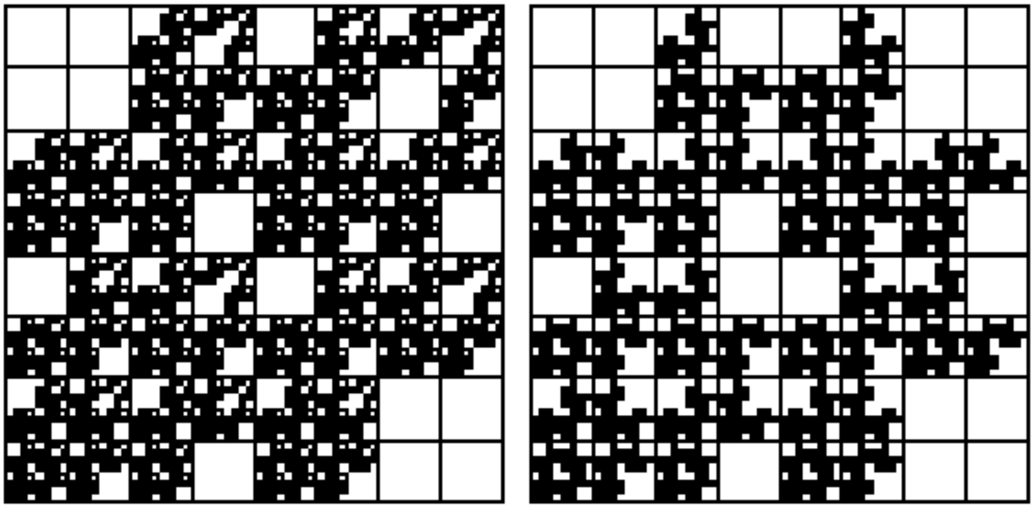
$$d(A) = \lim_{r \to 0} \text{Log}(N_r(A))/\text{Log}(1/r) = \lim_{r \to 0} \text{Log}(N_r(B)^2)/\text{Log}(1/r)$$

$$= \lim_{r \to 0} 2*\text{Log}(N_r(B))/\text{Log}(1/r) = 2*d(B).$$

(c) Recall d(A U B) = max{d(A), d(B)}. Both A and B are gaskets made of N = 3 pieces each scaled by r = 1/2, so d(A) = d(B) = Log(3)/Log(2). Consequently,d(A U B) = Log(3)/Log(2).



3. For (i) the forbidden pairs are 22 and 33; for (ii) the forbidden pairs are 22, 33, and 44.

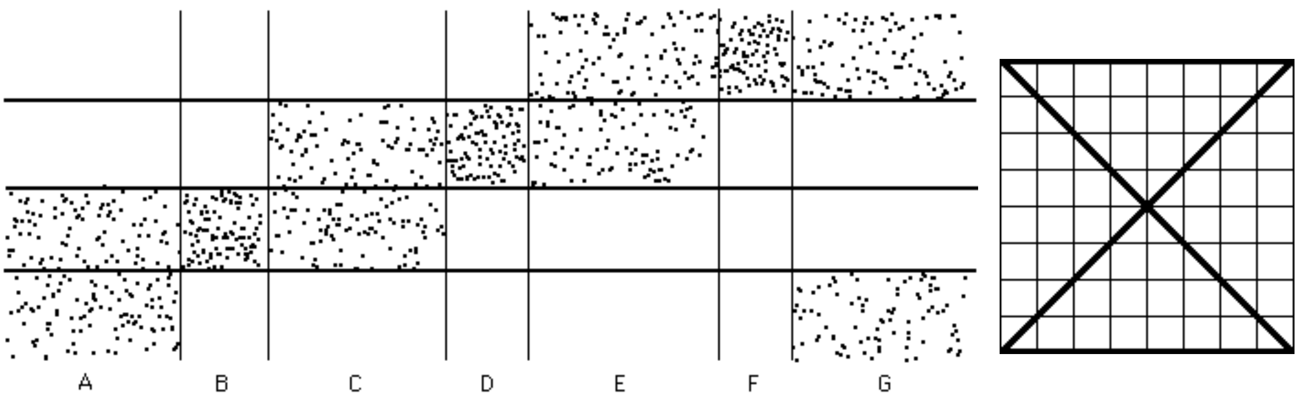


For (i) in addition to 22* and 33*, the forbidden triples are 122, 133, 233, 322, 422, 433, and 441.

For (ii) in addition to 22*, 33*, and 44*, the forbidden triples are 122, 133, 144, 233, 244, 322, 344, 433, and 422.

Note that (i) has a forbidden triple, 441, that does not contain a forbidden pair, so (i) is not generated by forbidden pairs.

4. (a) Here is one time series that generates the driven IFS picture.



Here is a description of how the time series generates the driven IFS.

A: points lie in bins 1 and 2; the driven IFS points lie on the line connecting corners 1 and 2.

B: points lie in bin 2; the driven IFS points move to corner 2.

C: points lie in bins 2 and 3; the driven IFS points lie on the line connecting corners 2 and 3.

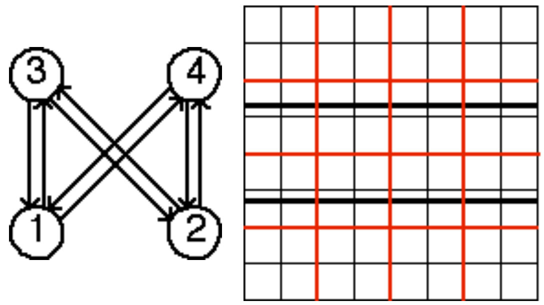
D: points lie in bin 3; the driven IFS points move to corner 3.

E: points lie in bins 3 and 4; the driven IFS points lie on the line connecting corners 3 and 4.

F: points lie in bin 4; the driven IFS points move to corner 4.

G: points lie in bins 1 and 4; the driven IFS points lie on the line connecting corners 1 and 4.

(b) The graph on the left generates the driven IFS on the right.



Note the occupied length 2 address squares on the right are 13, 14, 23, 24, 31, 32, 41, and 42.

To guarantee those squares are occupied, we need these arrows

3 -> 1, 4 -> 1, 3 -> 2, 4 -> 2, 1 -> 3, 2 -> 3, 1 -> 4, and 2 -> 4.

5. (a) Recall that if all the scaling factors r_i have the same value, α takes the form

$$\alpha(q) = (p_1{}^q \text{Log}(p_1) + \ldots + p_N{}^q \text{Log}(p_N)) / (\text{Log}(r)\,(p_1{}^q + \ldots + p_N{}^q))$$

With each $p_i = 1/N$, this becomes

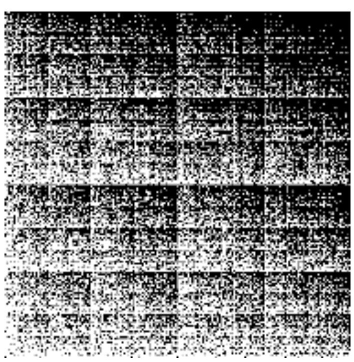
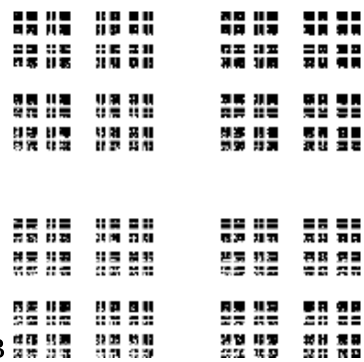
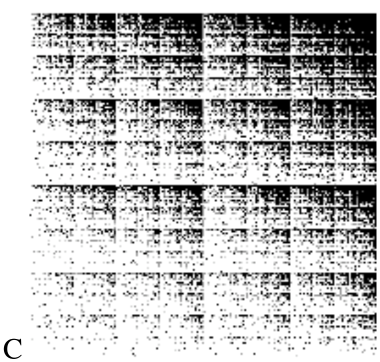
$$\alpha(q) = (N((1/N)^q \text{Log}(1/N))) / (\text{Log}(r)\, N((1/N)^q)) = \text{Log}(1/N)/\text{Log}(r) = \text{Log}(N)/\text{Log}(1/r)$$

(b) A corresponds to D, B correpsonds to F, C corresponds to E

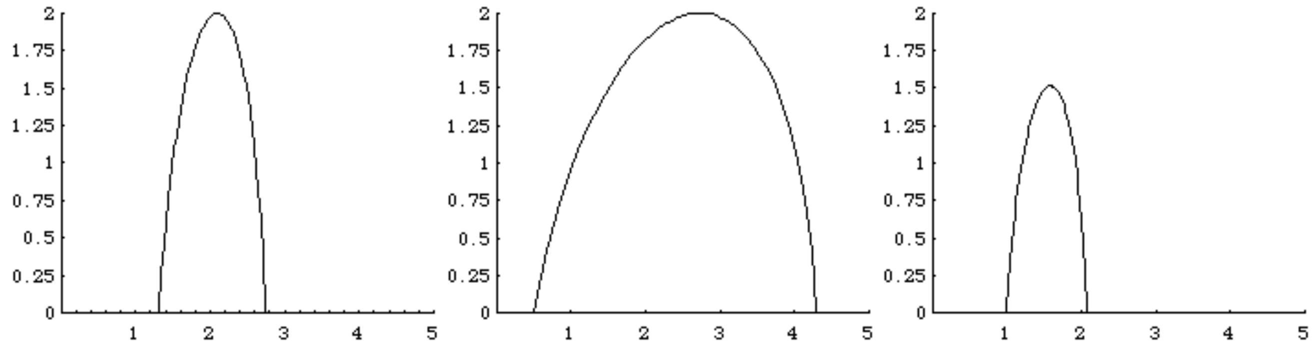
A is more uniform and has as attractor the unit square, hence of dimension 2. Consequently, the f(α) curve has maximum height 2 and and a narrow range of α values. This is the f(α) curve D.

B has attractor a Cantor set of dimension less than 2. Consequently, the f(α) curve has maximum height less than 2. This is the f(α) curve F.

C is less uniform and has as attractor the unit square, hence of dimension 2. Consequently, the f(α) curve has maximum height 2 and and a wide range of α values. This is the f(α) curve E.

A  B  C 

D E F


2
1.75
1.5
1.25
1
0.75
0.5
0.25
0
1 2 3 4 5