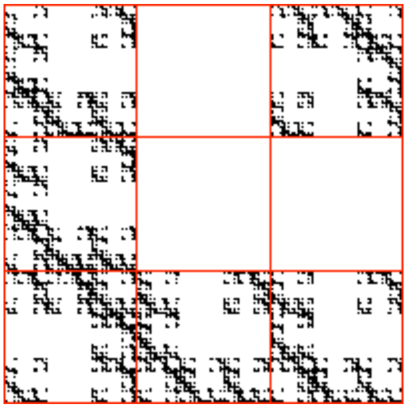
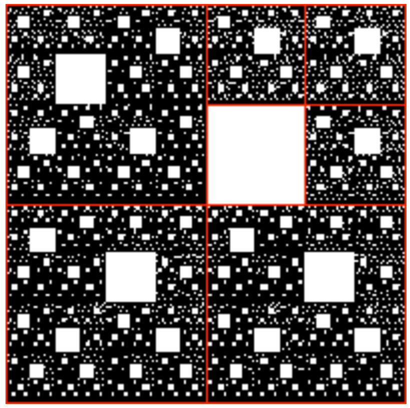
Practice Exam 3 Solutions

1. Here are IFS rules to generate each of these fractals.



R	S	θ	φ	E	F
0.333	0.333	180	180	0.333	0.333
0.333	0.333	0	0	0.333	0.0
0.333	0.333	0	0	0.667	0.0
0.333	0.333	0	0	0.0	0.333
0.333	0.333	0	0	0.0	0.667
0.333	0.333	180	180	1.0	1.0



R	S	θ	φ	E	F
0.5	0.5	0	0	0.0	0.0
0.5	0.5	0	0	0.5	0.0
-0.5	0.5	0	0	0.5	0.5
0.25	0.25	0	0	0.5	0.75
0.25	0.25	0	0	0.75	0.5
0.25	0.25	0	0	0.75	0.75

2. Fractal (a) consists of N=6 pieces each scaled by a factor of r = 1/3. Consequently, the similarity dimension is d = Log(6)/Log(3).

Fractal (b) consists of 3 pieces scaled by 1/2 and 3 pieces scaled by 1/4. The similarity dimension, d, is given by the Moran equation $3\cdot((1/2)^d) + 3\cdot((1/4)^d) = 1$.

Taking $x = (1/2)^d$, the Moran equation becomes

$$3x + 3x^2 = 1$$

The positive solution is $x = (-3 + \sqrt{(21)})/6$, and so

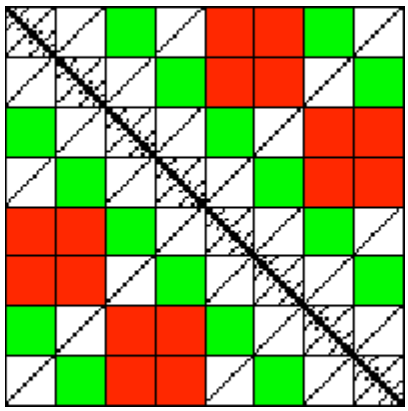
$$d = \text{Log}((-3 + \sqrt{(21)})/6)/\text{Log}(1/2).$$

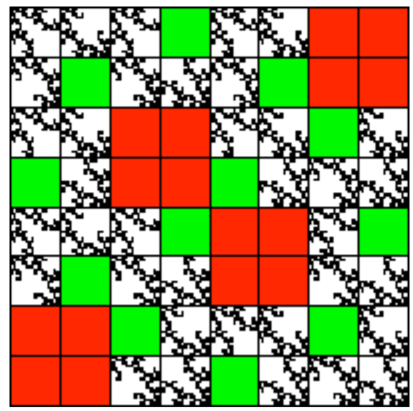
3. (a) The right image has $9 = 3^2$ copies. This can be achieved only if the first image gives rise to 3 copies, each of which in turn gives rise to 3 copies. Consequently, the smallest number of transformations is 3.

(b) From the placement of the squares, the lower left and upper rules involve some change of orientation. Note that reflection across either the x- or the y-axis keeps the long side of the L vertical. Consequently, these must involve rotations. From the placements of the three small squares in the lower left and upper corners, we see the lower left involves a counterclockwise rotation, and the upper left a clockwise rotation. Adjusting the translations to give the correct placements, we see

R	S	θ	φ	E	F
0.5	0.5	90	90	0.5	0.0
0.5	0.5	0	0	0.5	0.0
0.5	0.5	-90	-90	0.0	1.0

4. Empty length 2 address squares are filled in **red**, empty length 3 address squares are filled in **green**

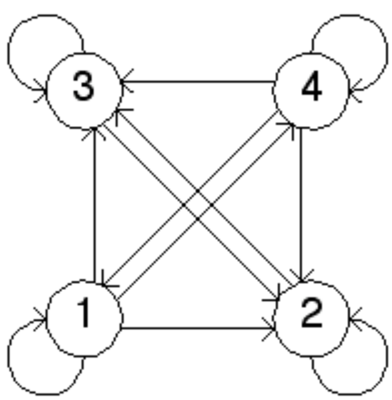


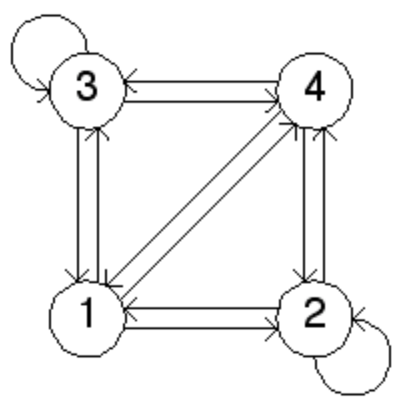


(a) In the left image, the empty length 2 addresses are 12, 13, 42, and 43. The empty length 3 addresses are 112, 113, 142, 143, 212, 213, 242, 243, 312, 313, 342, 343, 412, 413, 442, and 443. Each contains an empty length 2 address, so at least through length 3 addresses, this shape is determined by forbidden pairs.

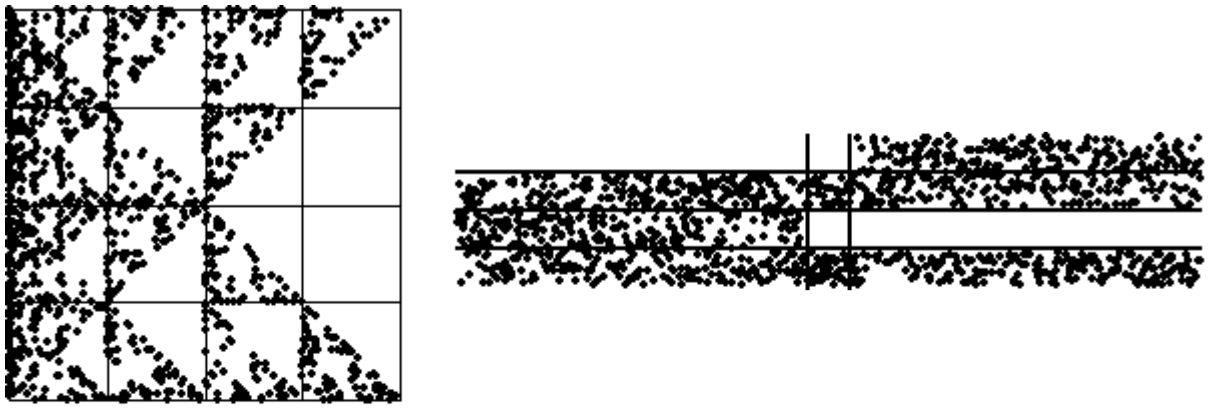
In the right image, the empty length 2 addresses are 11, 23, 32, and 44. The empty length 3 addresses are 123, 132, 144, 211, 223, 244, 311, 332, 344, 411, 423, and 432. Each contains an empty length 2 address, so at least through length 3 addresses, this shape is determined by forbidden pairs.

(b) Here are the corresponding graphs.



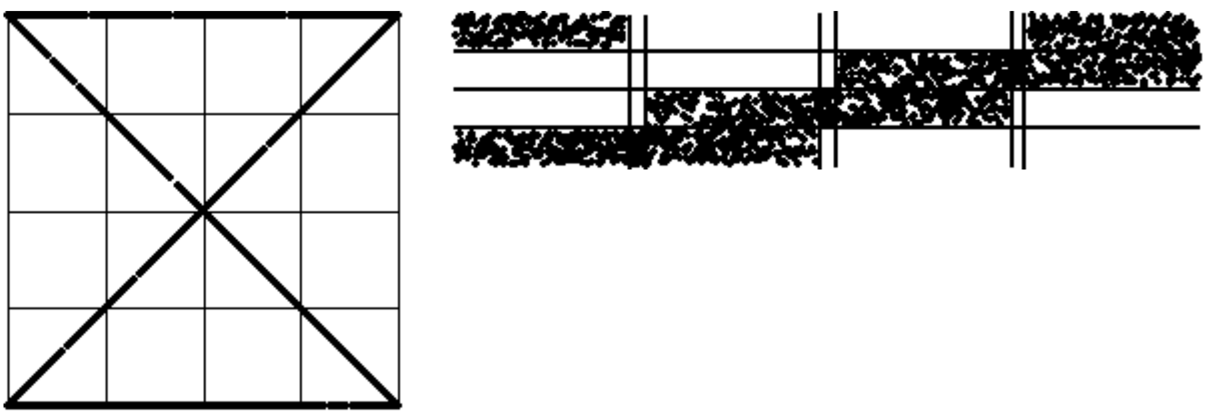


5. Here are the driven IFS and a time series that generates each.



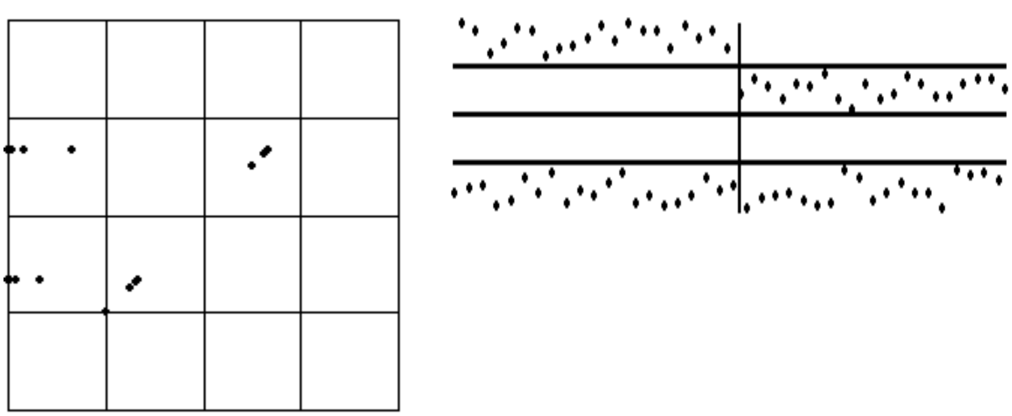
(a)

We note two gaskets, one between corners 1, 2, and 3, the other between corners 1, 3, and 4. To make a clean pair of gaskets, we must have a sector of the time series producing points in the line between corners 1 and 3, the set common to those gaskets. The left portion of the time series shown generates the 1, 2, 3 gasket. The center portion generates the 1, 3 line. The right portion generates the 1, 3, 4 gasket.



(b)

Here we need four line segments, together with portions of the time series producing the common part of those lines. The time series shows the 1-4 line, then moving to corner 1, the 1-2 line, then moving to corner 2, the 2-3 line, then moving to corner 3, then the 3-4 line.



(c)

Here we notice two things, points convering to the 1-4 cycle, and points converging to the 1-3 cycle. The time series begins with a cycle between bins 1 and 4, then with a cycle between bins 1 and 3.

6. (a) The Cantor middle halves set C is made of N = 2 pieces, each scaled by r = 1/4, so its similarity dimension is Log(2)/Log(4) = 1/2.

(b) By the product formula, the product of two Cantor middle halves sets has dimension

$$\dim(C \times C) = \dim(C) + \dim(C) = 1/2 + 1/2 = 1$$

(c) By the intersection formula, the typical intersection of the product C × C and a line segment A is

$$\dim((C \times C) \text{ intersect } A) = \dim(C \times C) + \dim(A) - 2 = 1 + 1 - 2 = 0$$

(d) The horizontal line intersects C × C in a copy of the Cantor set C, hence having dimension 1/2. The diagonal line intersects the product C × C in one point, of dimension 0.