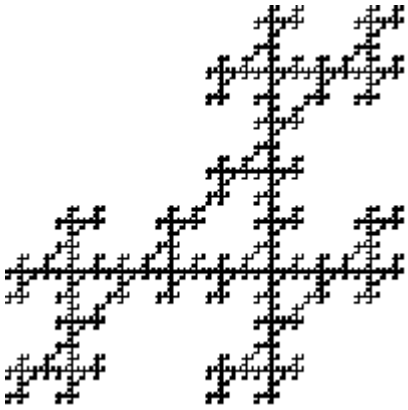
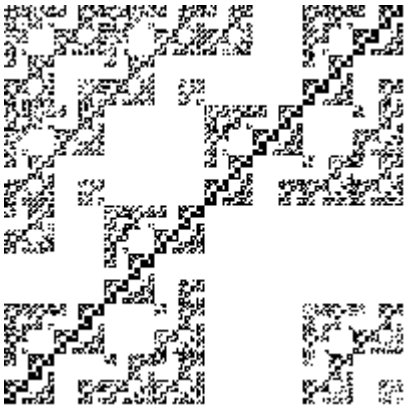
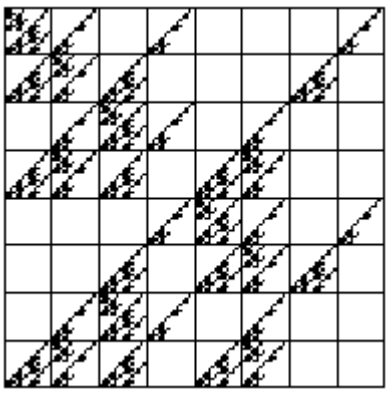
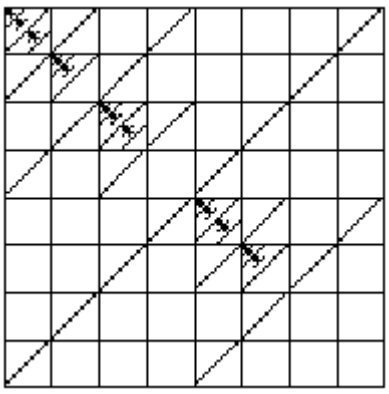
Practice Exam 4

1. Find IFS rules to generate these fractals.

(a)  (b) 

2. Find the similarity dimensions of the fractals (a) and (b) of problem 1. If the Moran equation is used, solve it exactly using the quadratic formula.

3. (a) Pictured below are two IFS with memory images. Determine if either can be generated by forbidden pairs. Explain how you arrived at our answer. Give explicit details. For reference, the length three address squares are shown on both images.

(a)  (b) 

(b) For each image that is generated by forbidden pairs, fill in the appropriate arrows on the corresponding graph.

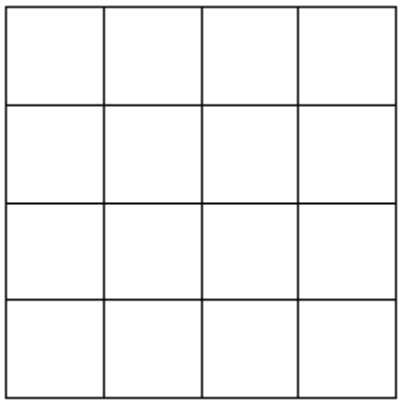
(a) 

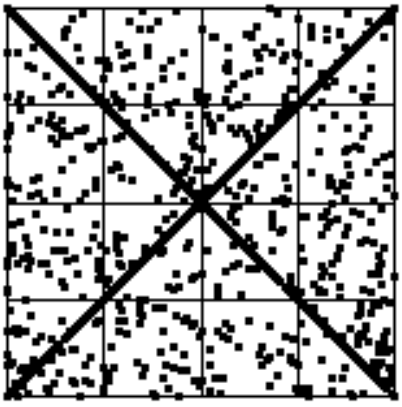
(b) 

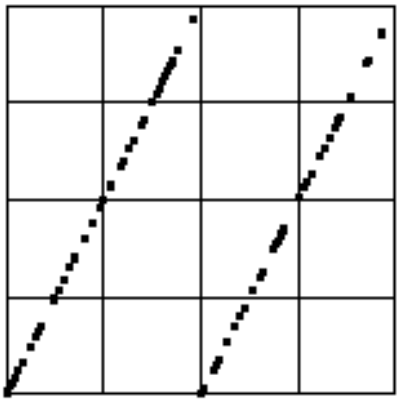
(c) Can either of these images be generated by IFS without memory? Explain how you arrived at our answer. Give explicit details.

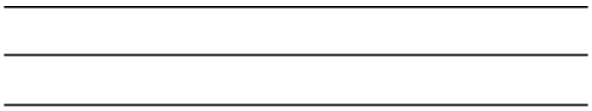
(d) If either of these images be generated by IFS without memory, can they be generated by a finite number of transformations? Explain how you arrived at our answer. Give explicit details. If your answer is "yes," list the transformations.

4. Pictured below are the driven IFS or the time series for three examples. If the driven IFS is given, sketch a time series that could generate that driven IFS. If the time series is given, sketch the driven IFS it would generate. Explain how you arrived at your answer. Provide enough detail in the time series or driven IFS to illustrate your explanation.

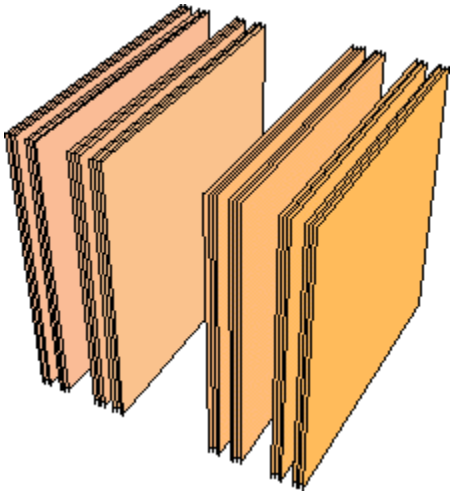








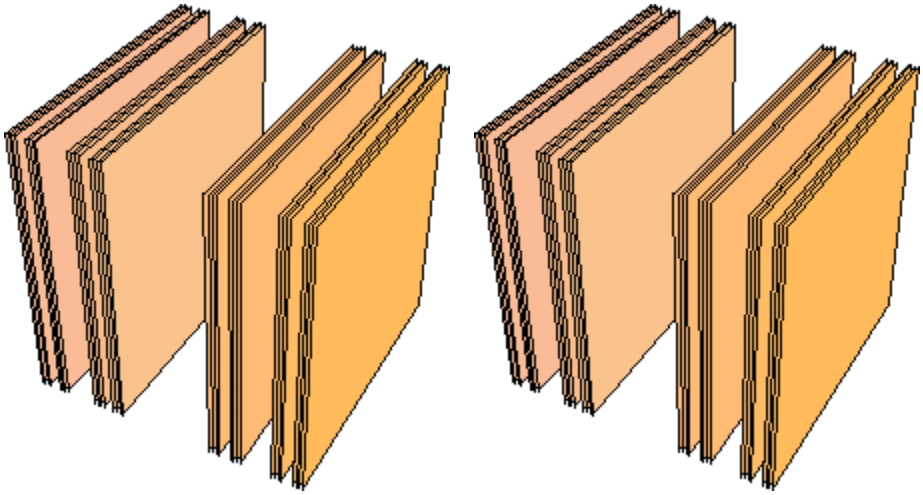
5. Pictured here is the product P of a Cantor middle-thirds set and a filled-in square.



(a) What is the dimension of this set P? Justify your answer.

(b) What is the typical dimension of the intersection P ∩ L of the set P and a line segment L? Both L and P lie in 3-dimensional space. Justify your answer.

(c)On the left, sketch a placement of L so that P ∩ L has dimension 1; on the right sketch a placement of L so that P ∩ L has dimension Log(2)/Log(3). Explain your choices.



6. Suppose $N_A(r)$ and $N_B(r)$ denote the number of boxes of side length r needed to cover the fractals A and B.

(a) Write the formula for the box-counting dimension d(A) of A.

(b) Find a relation between $N_A(r)$ and $N_B(r)$ so d(B) = (1/2)d(A).