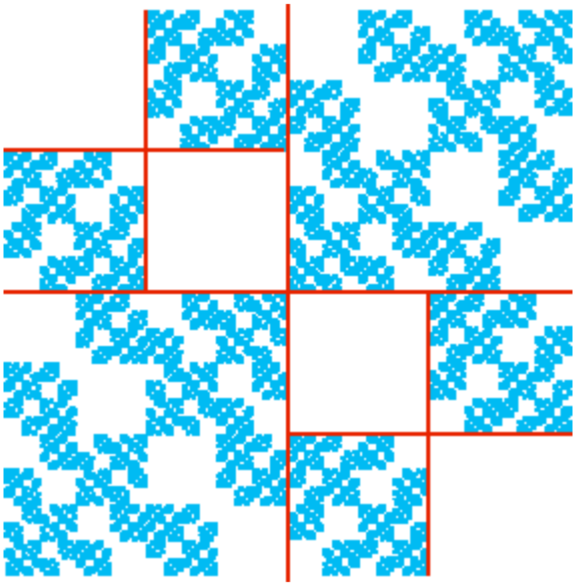
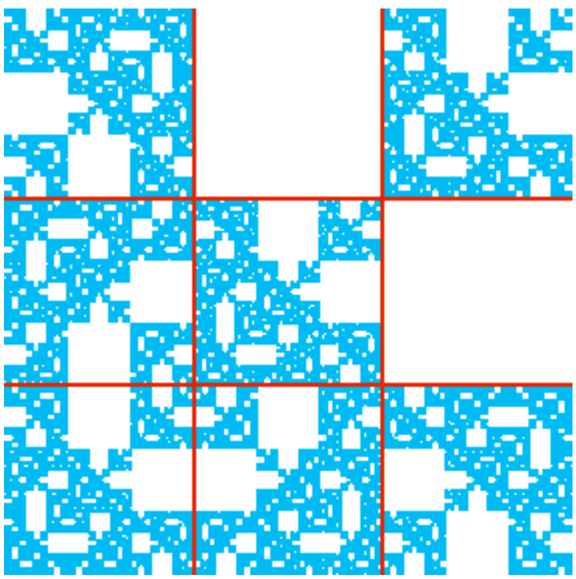
Practice Exam 5 Solutions

1. (a)



r	s	θ	φ	e	f
0.5	0.5	0	0	0	0
0.5	0.5	0	0	0.5	0.5
-0.25	0.25	0	0	0.75	0
-0.25	0.25	0	0	1	0.25
-0.25	0.25	0	0	0.25	0.5
-0.25	0.25	0	0	0.5	0.75

(b)



r	s	θ	φ	e	f
0.333	0.333	0	0	0	0
0.333	0.333	0	0	0.333	0.333
0.333	0.333	0	0	0.667	0.667
0.333	0.333	-90	-90	0	0.667
0.333	0.333	180	180	0.333	1.0
0.333	0.333	90	90	0.667	0
0.333	0.333	180	180	1	0.333

2. (a) There are two pieces scaled by 0.5, and four pieces scaled by 0.25. Taking $x = .5^d$, the Moran equation becomes

$$2x + 4x^2 = 1$$

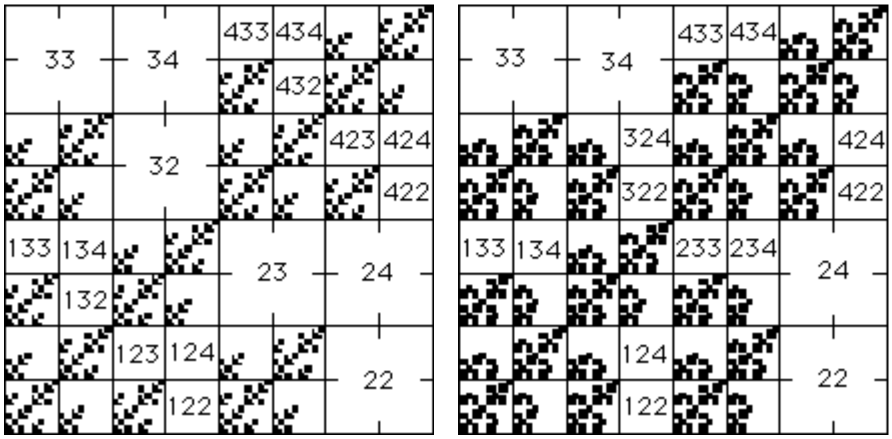
The positive solution is $(-1 + \sqrt{5})/4$, so $d = \text{Log}((-1 + \sqrt{5})/4)/\text{Log}(1/2)$.

(b) This fractal consists of N = 7 pieces, each scaled by r = 1/3, so the dimension is Log(7)/Log(3).

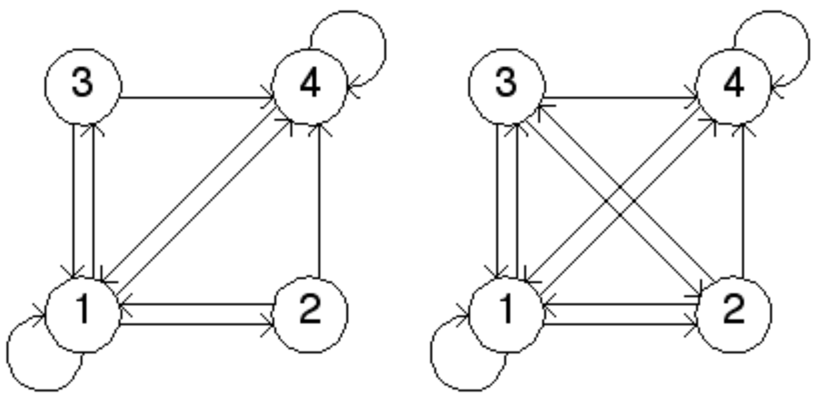
3. (a) For the left fractal, the forbidden pairs are 22, 23, 24, 32, 33, and 34. Each of the forbidden triples contains one of these forbidden pairs, so at least to the level of forbidden triples, this fractal is

generated by forbidden pairs.

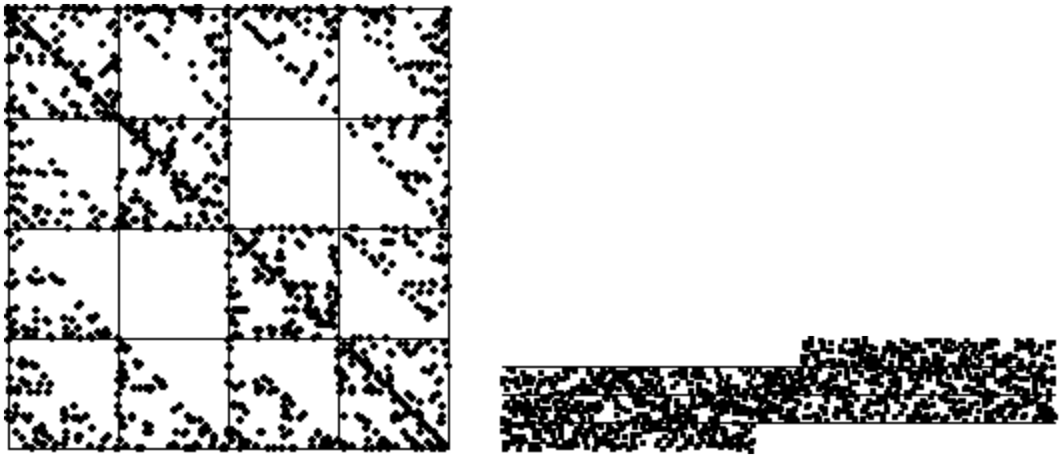
For the right fractal, the forbidden pairs are 22, 24, 33, and 34. Each of the forbidden triples contains one of these forbidden pairs, so at least to the level of forbidden triples, this fractal is generated by forbidden pairs.



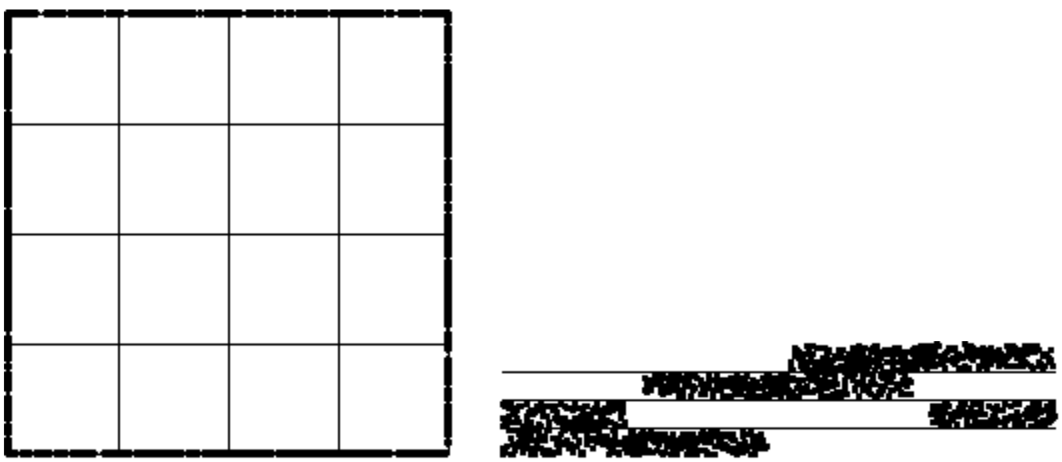
(b) Here are the graphs for these fractals.



4. (a) In the driven IFS we see two gaskets, one generated by T_1, T_2, and T_3, the other by T_2, T_3, and T_4. To produce the first, some part of the time series must consist of points scattered randomly between bins 1, 2, and 3; to produce the second, another part must consist of points scattered randomly between bins 2, 3, and 4. The driven IFS contains no points not on these gaskets, so the transition between gaskets must be achieved by a part of the time series that occupies bins common to both. We choose a part with points scattered randomly between bins 2 and 3, producing the slightly more fully occupied diagonal in both gaskets.



(b) The driven IFS consiste of four line segments. Because every point lies on one of these line segments, the transitions between line segments must be achieved by points in the common bin. One time series that would produce this driven IFS has regimes ordered in this way. Points scattered randomly between bins 1 and 2, points in bin 1, points scattered randomly between bins 1 and 3, points in bin 3, points scattered randomly between bins 3 and 4, points in bin 4, points scattered randomly between bins 4 and 2.

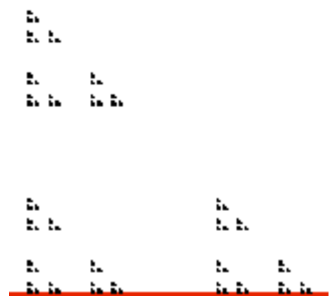


5. (a) This fractal consists of $N = 3$ pieces, each scaled by a factor of 1/3, so the dimension is $\log(3)/\log(3) = 1$.

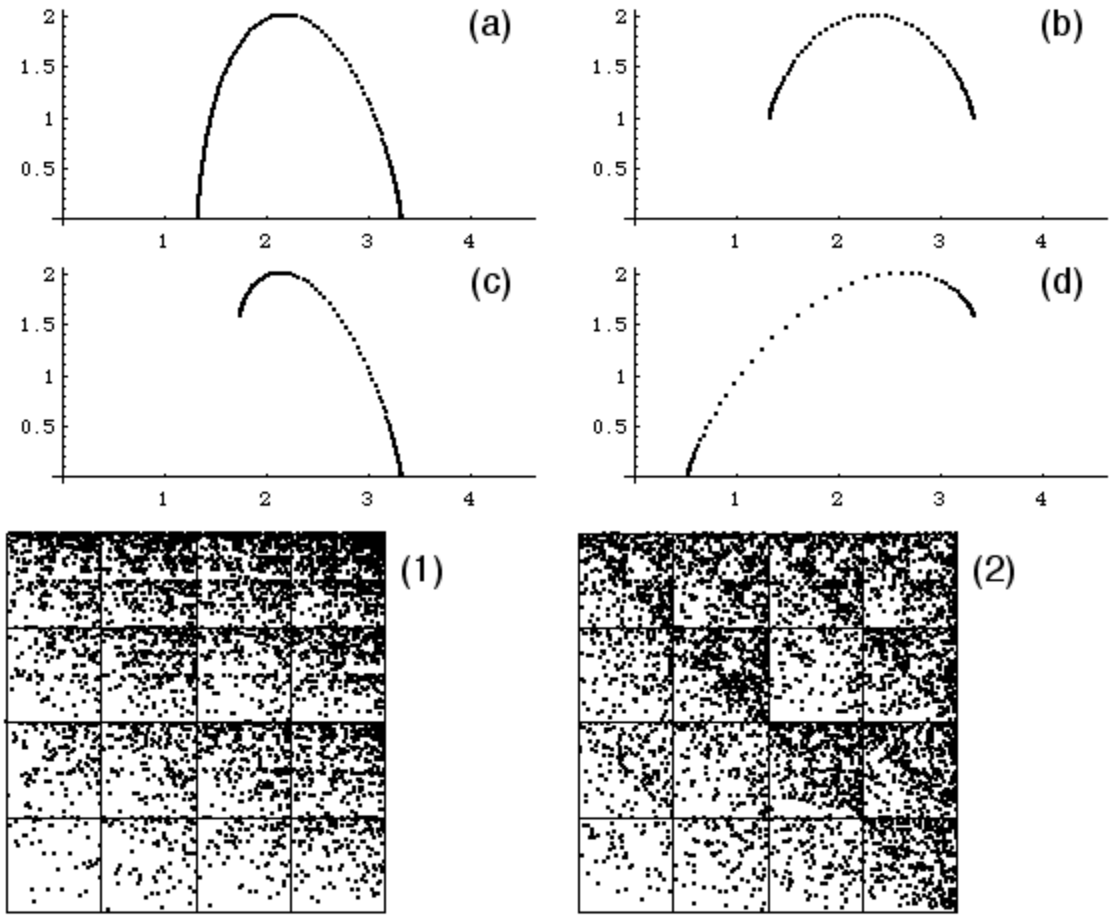
(b) By the intersection formula, the typical intersection of G and a line segment L is

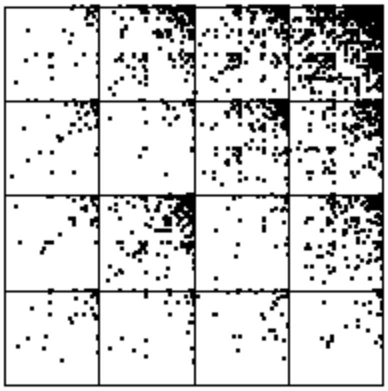
$$\dim(G \cap L) = \text{gim}(G) + \dim(L) - 2 = 1 + 1 - 2 = 0.$$

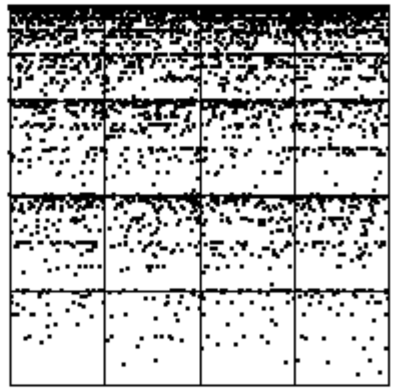
(c) Placing the line segment along the bottom of the gasket G, the intersection is a Cantor middle-thirds set, consisting of $N = 2$ pieces, each scaled by $r = 1/3$, and so having dimension $\log(2)/\log(3)$.



6. To match the f(α) curves with the driven IFS, from the relative densities of points in the driven IFS, we determine the probabilities of each of the transformations.




(3)


(4)

(1) It appears that prob 1 < prob 2 < prob 3 < prob 4, so for this driven IFS we have both min(α) and max(α) occur at single points. Then f(min(α)) = f(max(α)) = dim(point) = 0. That is, (a) is the f(α) curve for driven IFS (1).

(2) It appears that 2, 3, and 4 have about the same high probability, and 1 has the lowest probability. Then f(min(α)) = dim(gasket) = log(3)/log(2) ≈ 1.6, and f(max(α)) = dim(point) = 0. That is, (c) is the f(α) curve for the driven IFS (2).

(3) It appeas that 1, 2, and 3 have about the same low probability, and 4 has the highest probability. Then f(min(α)) = dim(pt) = 0 and f(max(α)) = dim(gasket) ≈ 1.6. That is, (d) is the f(α) curve for driven IFS (3).

(4) It appears that 3 and 4 have the same high probability, and 1 and 2 have the same low probability. Then f(max(α)) = f(min(α)) = dim(line) = 1. That is, (b) is the f(α) curve for driven IFS (4).