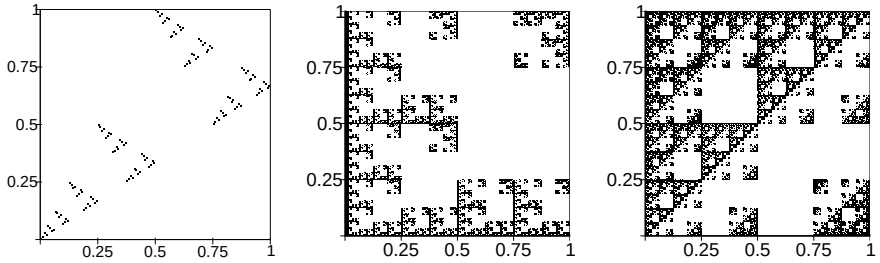
Final Exam

1. Find IFS rules to generate these fractals.



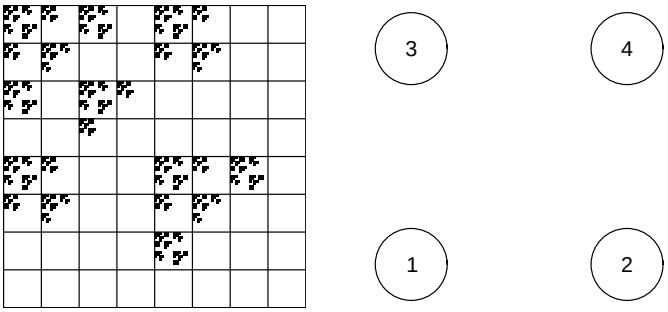
2. Compute the dimensions of each of the fractals in problem 1. If the Moran equation is needed, solve the equation using the quadratic formula, not numerically.

3. Suppose for each $n > 0$, the minimum number of boxes of side length $\epsilon = 1/2^n$ needed to cover a fractal A is

$$N(\epsilon) = 2^n + 3^n + 4^n$$

Compute the box-counting dimension of A.

4. (a) Show the IFS with memory shown on the left can be generated by forbidden pairs. On the right draw the transition graph that generates this IFS.



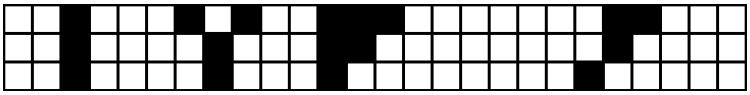
(b) Can this fractal be generated by an IFS without memory? Support your answer. If it can be, find the IFS rules.

(c) Write the equation to find the dimension of this fractal. You need not solve this equation.

5. Sketch the $f(\alpha)$ curve generated by this IFS. Label and put scales on the axes. Label the important points, and find the values of $\alpha_{\min}$, $\alpha_{\max}$, $f(\alpha_{\min})$, $f(\alpha_{\max})$, and the maximum value of $f(\alpha)$. Where appropriate, leave your answers as a ratio of logs; do not give decimal expressions. Give reasons to support your answers.

r	s	θ	φ	e	f	prob
.25	.25	0	0	0	0	0.05
.25	.25	0	0	.25	0	0.05
.25	.25	0	0	.25	.25	0.1
.25	.25	0	0	0	.25	0.1
.25	.25	0	0	.5	.5	0.1
.25	.25	0	0	.75	.75	0.2
.25	.25	0	0	.75	0	0.2
.25	.25	0	0	0	.75	0.2

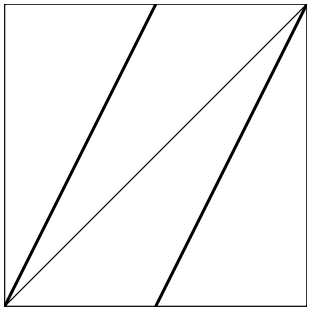
6. Can this be three successive generations of an $N = 3$ binary CA? Give the rule if you believe there is one; give your reasoning if you think there is none.



7. Suppose a disc A is attached to a disc B attached to the main cardioid of the Mandelbrot set, and A has cycle number 18. List all the possible cycle numbers b of B and explain how you arrived at your answer.

8. Pictured here is the graph of a function f inside the unit square, together with the line $y = x$. The function is defined by

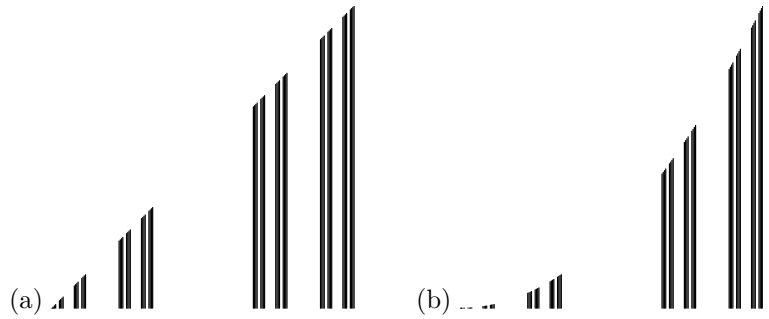
$$f(x) = \begin{cases} 2x & \text{for } x \leq 1/2 \\ 2x - 1 & \text{for } x > 1/2 \end{cases}$$



(a) On the graph sketch the graphical iteration plot of a 2-cycle.

(b) Using the formula for the function f, find the values x_1 and x_2 of the 2-cycle.

9. Here we consider the portion of the product of a Cantor middle thirds set along the x-axis, and the unit interval along the y-axis, in (a) the portion lying under the line $y = x$, in (b) the portion lying under the parabola $y = x^2$. Find the dimensions of these fractals. Give the exact answer, not a decimal approximation.



(a) (b)