Dec 3 Reviews

Mandelbrot set and Julia sets

iteration $z_{n+1} = z_n^2 + c$

writing $z_n = x_n + i y_n$

$z_{n+1} = x_{n+1} + i y_{n+1}$

$c = a + ib$

$$x_{n+1} + i y_{n+1} = (x_n + i y_n)^2 + (a+ib)$$

$$= x_n^2 - y_n^2 + 2 x_n y_n i + a + ib$$

equivalent real-number iteration

$$x_{n+1} = x_n^2 - y_n^2 + a$$

$$y_{n+1} = 2 x_n y_n + b.$$

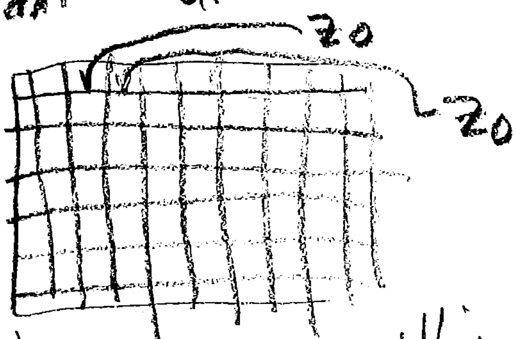
If the distance of z_n to the origin is greater than 2, the later iterates, z_{n+1}, z_{n+2}, z_{n+3}, ... run away to infinity.

For each c, the Julia set of c is the collection of all points, label each as z_0, for which

$$z_1 = z_0^2 + c$$
$$z_2 = z_1^2 + c$$
$$\vdots$$

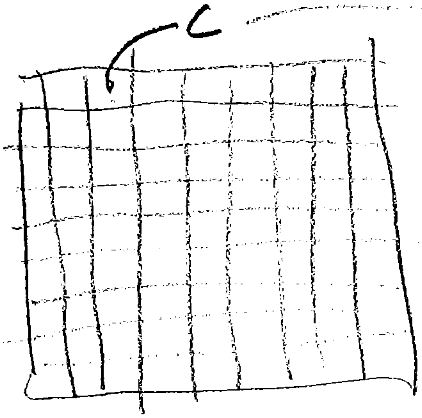
do not run away to ∞, that is, they don't get farther than 2 from the origin.

c is constant - different Julia set for each c.



If all iterates (we are willing to try) stay within 2 of the origin, we say z_0 belongs to the Julia set and we paint the pixel black. Otherwise, z_0 does not belong to the Julia set and we paint the pixel a color corresponding to how many steps it took to get farther than 2 from the origin.

For the Mandelbrot set, always start the iterations with $z_0 = 0$



for this c, start with $z_0 = 0$, then
$z_1 = z_0^2 + c$,
$z_2 = z_1^2 + c$

If the iterates stay within 2 of the origin, c belongs to the Mandelbrot set and we paint the pixel black.
If some iterate gets farther than 2 from the origin, c does not belong to the Mandelbrot set, and we paint the pixel a color based on when the iterates first get larger than 2

Reason: Theorems of Fatou & Julia

<u>Theorem 1</u> For z^2+c, Julia sets are either connected or are Cantor sets

<u>Theorem 2</u> For z^2+c, the Julia set is connected if and only if the iterates of $z_0 = 0$ do not run away to infinity.

Then the Mandelbrot set, all c values for which the iterates of $z_0 = 0$ do not run away to infinity, is the collection of c for which the Julia sets are connected.