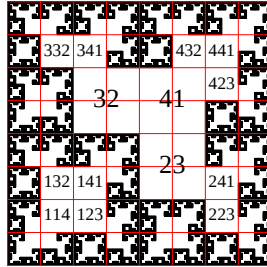
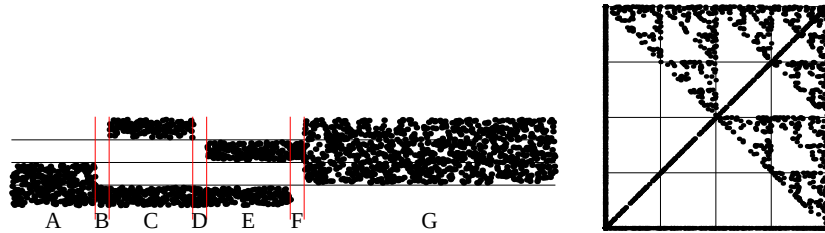


Third homework set solutions

1. The forbidden pairs are 23, 32, and 41. The forbidden triples are 114, 123, 132, 141, 223, 241, 332, 341, 423, 432, and 441. The forbidden triple 114 does not contain any forbidden pair 23, 32, or 41, so this IFS cannot be generated by 1-step memory.



2. The time series splits into seven regimes, A with points in bins 1 and 2, B with points in bin 1, C with points in bins 1 and 4, D with points in bin 1, E with points in bins 1 and 3, F with points in bin 3, and G with points in bins 2, 3, and 4.



Time series points in regime A generate driven IFS points along the line between $(0,0)$ and $(1,0)$.

Time series points in regime B generate driven IFS points converging to the point $(0,0)$.

Time series points in regime C generate driven IFS points along the line between $(0,0)$ and $(1,1)$.

Time series points in regime D generate driven IFS points converging to the point $(0,0)$.

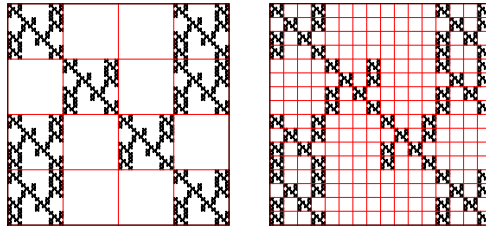
Time series points in regime E generate driven IFS points along the line between $(0,0)$ and $(0,1)$.

Time series points in regime F generate driven IFS points converging to the point $(0,1)$.

Time series points in regime G generate driven IFS points on the gasket with vertices $(1,0)$, $(0,1)$, and $(1,1)$.

3. In the left image, we see $N(1/4) = 8$; on the right we see $N(1/16) = 64$. In general, we have

$$N(1/4^n) = 8^n$$



Then the box-counting dimension is

$$\begin{aligned} d_b &= \lim_{n \rightarrow \infty} \frac{\log(N(1/4^n))}{\log(1/(1/4^n))} \\ &= \lim_{n \rightarrow \infty} \frac{\log(8^n)}{\log(4^n)} \\ &= \lim_{n \rightarrow \infty} \frac{n \log(8)}{n \log(4)} \\ &= \frac{\log(8)}{\log(4)} = 1.5. \end{aligned}$$