Fifth homework set

Due at the beginning of class on Thursday, Oct 15. No late homework will be accepted.
Fold your homework paper vertically and PRINT your name on the outside.

1. Suppose C is a Cantor middle-halves set, that is, a Cantor set consisting of $N = 2$ pieces scaled by $r = 1/4$. In parts (a), (b), and (c) give exact values, not decimal approximations.
(a) Find the dimension of C. Your answer should be a simple fraction, and not contain logs.

(b) Find the dimension of the product $C \times C \times C$.

(c) Suppose L is a line segment. Find the dimension of the typical intersection of L and $C \times C \times C$. Assume both L and $C \times C \times C$ lie in 3-dimensional space.

(d) Describe a placement of L so $L \cap (C \times C \times C)$ has dimension $1/2$.

2. Suppose C is the Cantor middle-halves set of problem 1, and S is a filled-in unit square.

(a) Find the exact value for the typical dimension of $S \cap (C \times C \times C)$. Assume both S and $C \times C \times C$ lie in 3-dimensional space.
(b) Describe a placement of S so $S \cap (C \times C \times C)$ has dimension 1.

3. Consider the multifractal generated by this IFS. For each of part (a) through (e), explain how you arrived at your answer. For (a) and (b), express your answer to three places to the right of the decimal. For (c), (d), and (e) give exact values, not decimals.

transformation	r	s	θ	φ	e	f	prob
1	1/2	1/2	0	0	0	0	.025
2	1/4	1/4	0	0	1/2	0	.15
3	1/4	1/4	0	0	3/4	0	.15
4	1/4	1/4	0	0	1/2	1/4	.15
5	1/4	1/4	0	0	3/4	1/4	.15
6	1/4	1/4	0	0	1/2	1/2	.15
7	1/8	1/8	0	0	0	7/8	.075
8	1/8	1/8	0	0	1/2	7/8	.075
9	1/8	1/8	0	0	7/8	7/8	.075

(a) Find the maximum value of α.

(a) Find the minimum value of α.

(c) Find the exact value of $f(\alpha_{\max})$.

(d) Find the exact value of $f(\alpha_{\min})$.

(e) Find the exact maximum value of $f(\alpha)$. Hint: remember division of polynomials.