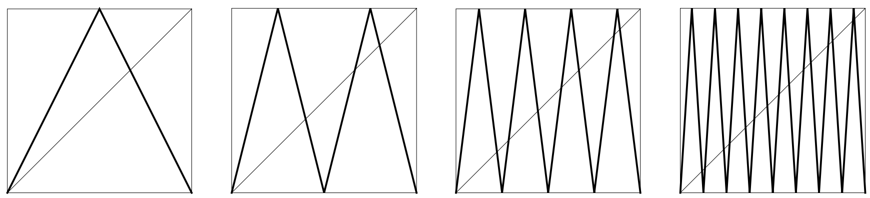
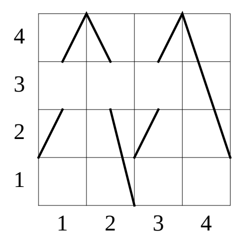
Seventh homework set solutions

1. As we see in the graphs, T has 2 fixed points, T^2 has 4 fixed points, T^3 has 8 fixed points, and T^4 has 16 fixed points. Of these, 2 are fixed points for T, so also are fixed by T^4, and 2 constitute a 2-cycle for T, so also are fixed by T^4. This leaves 12 fixed points for T^4 that are not fixed by T or T^2, so must make up 4-cycles for T. A 4-cycle contains 4 points, so these 12 fixed points for T^4 must constitute three 4-cycles for T. Because the slopes of the banches of the graph of T^4 have absolute values greater than 1 (in fact, the slopes are ± 16), these 4-cycles are unstable.

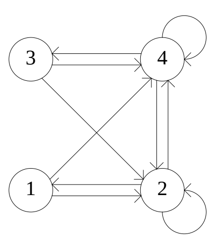


2. (a) From the graph we see the bin to bin transitions are

$$1 \to 2, 4;\ \ 2 \to 1, 2, 4;\ \ 3 \to 2, 4;\ \ 4 \to 2, 3, 4$$



(b) From the allowed transitions, we see the transition graph is



(c) Inspecting the transition graph, we see that 2 and 4 are romes. These give two copies of the fractal, scaled by 1/2. The other transitions, $2 \to 1$ and $4 \to 3$, produce two copies of the fractal scaled by 1/4. This is all. Then the dimension of the driven IFS fractal can be found by the Moran equation

$$2(1/2)^d + 2(1/4)^d = 1$$

Taking $x = (1/2)^d$, we have

$$2x + 2x^2 = 1$$

The positive solution is $x = (-1 + \sqrt{3})/2$, and so the dimension is

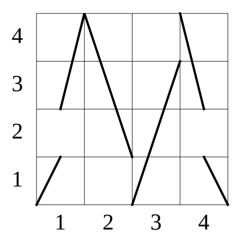
$$d = \log((-1 + \sqrt{3})/2)/\log(1/2)$$

3. From the graph we see the allowed transitions are

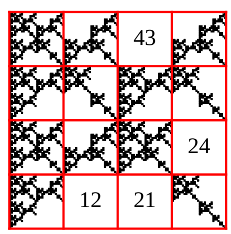
$$1 \to 1, 3, 4;\ \ 2 \to 2, 3, 4;\ \ 3 \to 1, 2, 3;\ \ 4 \to 1, 3, 4$$

and so the forbidden transitions are $1 \to 2$, $2 \to 1$, $3 \to 4$, and $4 \to 2$.

From the driven IFS, we see the empty length-2 addresses are 12, 21, 24, and 43 for (a), and 12, 21, 34, and 42 for (b). The forbidden transition of the graph produce these empty length-2 addresses 21, 12, 43, and 24, so driven IFS (a) corresponds to the graph.



(a)



(b)