

Eighth homework set solution

1. The neighborhood of cell A is DDD . Because the second generation cell below A is dead, we see $DDD \rightarrow D$ must hold for every CA giving the second row from the first. Continuing in this way, we find

$$\begin{aligned} &DDD \rightarrow D, DDL \rightarrow L, DLL \rightarrow D, LLD \rightarrow L, LDD \rightarrow L, \\ &DDL \rightarrow L, DLD \rightarrow D, LDL \rightarrow D, DLD \rightarrow D, LDD \rightarrow L \end{aligned}$$

Because of wraparound, the two unlabeled cells in the first generation both give $DDD \rightarrow D$. There are no inconsistencies, that is, we have no instances of a neighborhood configuration giving two different results, so there is an $N = 3$ binary CA that produces the second row from the first.

	A	B	C	D	E	F	G	H	I	J	

To see how many such CA, note that only one configuration, LLL , is unspecified. Consequently, there are two CA

$$DDL, LLD, LDD \text{ give } L, \text{ all others give } D$$

and

$$DDL, LLD, LDD, LLL \text{ give } L, \text{ all others give } D$$

that produce the second row from the first.

2. The initial generation contains only two neighborhood configurations: A D cell surrounded by four L cells, and an L cell surrounded by four D cells. A L cell surrounded by four D cells has only one L cell, itself, in its five cell neighborhood, so by the CA rule, this L cell dies. Also, a D cell surrounded by four L cells has four L cells in its neighborhood, so by the CA rule, this D cell becomes alive. Consequently, the second generation will have L cell everywhere the initial generation had a D cell, and will have a D cell everywhere the initial generation had a L cell. Note that in the second generation, every L cell is surrounded by four D cells, and every D cell is surrounded by four L cells. Consequently, this interchange of L and D cells continues forever.

3. First, we must understand the identity CA. In terms of neighborhoods, that the identity CA leaves every configuration unchanged means that a L central cell remains L regardless of its two neighbors, and a D central cell remains D regardless of its two neighbors. That is, this is the rule for the identity CA

$$LLL, LLD, DLL, DLD \text{ give } L, \text{ the other four configurations give } D$$

That is, the classifier system for the identity CA is this.



Then we see that crossing at e , the child consisting of the left piece of A and the right piece of B has the classifier system for the identity CA.

