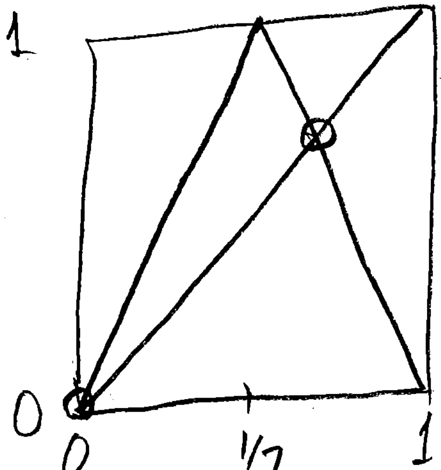
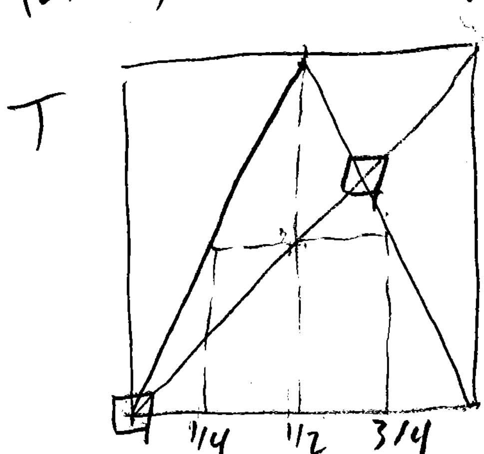
Fixed points and cycles.

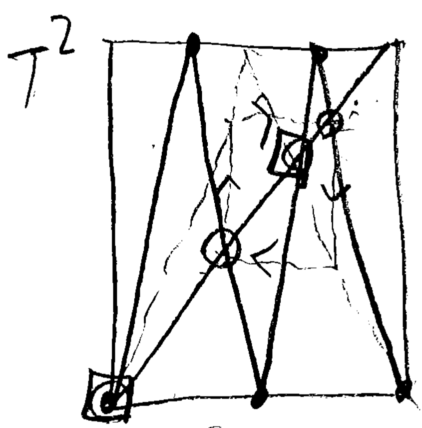
Tent map $T(x) = \begin{cases} 2x & \text{if } x \le 1/2 \\ 2-2x & \text{if } x \geq 1/2 \end{cases}$



The fixed points are the intersections of the graph of $T(x)$ and the diagonal line.

2-cycle points for T are fixed points for $T^2 = T(T)$.





T^2 has 4 fixed points

$T^2(0) = T(T(0)) = T(0) = 0$

$T^2(1) = T(T(1)) = T(0) = 0$

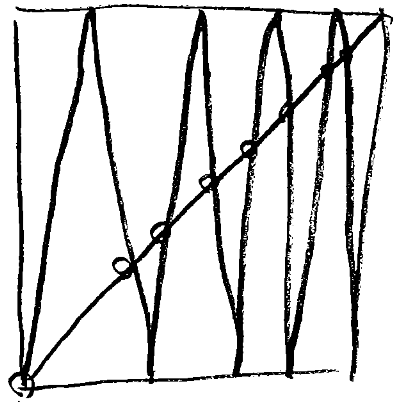
$T^2(1/2) = T(T(1/2)) = T(1) = 0$

$T^2(1/4) = T(T(1/4)) = T(1/2) = 1$

$T^2(3/4) = T(T(3/4)) = T(1/2) = 1$

Two are fixed points for T, so $4-2=2$ belong to a 2-cycle for T. T has one 2-cycle

T^3



T^3 has 8 fixed points
Two of these 8 are fixed points for T
This leaves $8-2=6$ points arranged in 3-cycles for T, so T has 2 3-cycles.

T has 2 fixed points
T^2 has $4 = 2^2$ fixed points
T^3 has $8 = 2^3$ fixed points
T^4 has $16 = 2^4$ fixed points
2 of the 16 are fixed points for T

$$\underbrace{16 - 2}_{\text{fixed points for } T} \underbrace{- 2}_{\text{2-cycle points}} = 12 \qquad T \text{ has 3 4-cycles}$$

T^7 has $2^7 = 128$ fixed points

$$128 - \underbrace{2}_{\text{fixed points for } T} = 126 \text{ points for 7-cycles}$$

18 7-cycles

p is prime, $2^p - 2$ is a multiple of p

Return maps

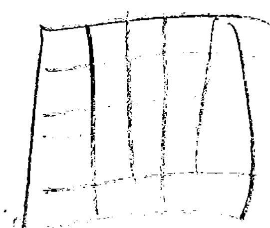
$x_1, x_2, x_3, x_4, \ldots$

plot $(x_1, x_2), (x_2, x_3), (x_3, x_4), \ldots$

If $x_{n+1} = L(x_n)$, then the return map points lie on the graph of L

Driven IFS - detecting allowed and forbidden combinations - not good for detecting cycles.

Kelly Plots = bin the data, assign a color to each bin, and plot the colors l to r, t to b.



good for detecting cycles, and also cycles mixed with chaos.