

Fractals: complicated shapes may have simple causes.

Multifractals: complicated distributions may have simple causes.

Chaos: complicated behaviors may have simple causes.

1976 - Robert May - simple population model with extremely complex behavior.

Tent map $x_{n+1} = T(x_n) = \begin{cases} 5 \cdot x_n & 0 \leq x_n \leq \frac{1}{2} \\ 5 - 5 \cdot x_n & \frac{1}{2} \leq x_n \leq 1 \end{cases}$

Logistic map - growth with limited resources

no limitation

$$P_{n+1} = (1 + b - d) P_n$$

per capita birth rate
per capita death rate

$$= r P_n$$

with limited resources - K = carrying capacity
= max. population the environment supports

$$P_{n+1} = r \left(1 - \frac{P_n}{K} \right) P_n$$

growth rate decreases as the population increases

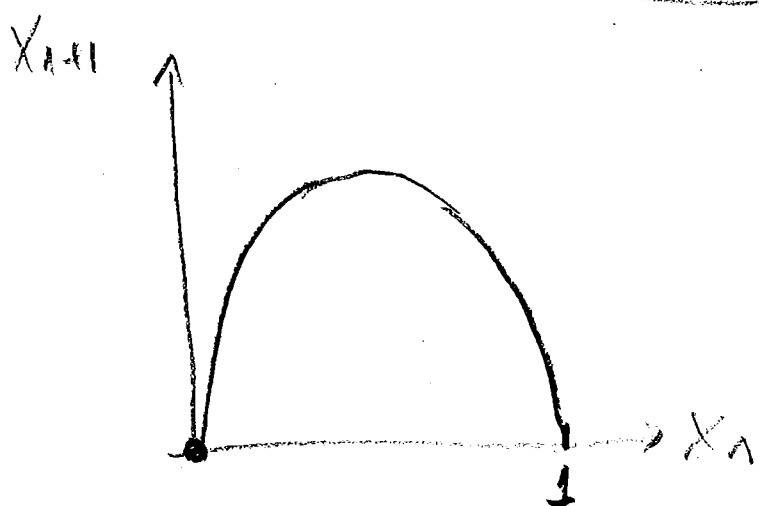
Write $x_n = \frac{P_n}{K}$, $x_{n+1} = \frac{P_{n+1}}{K}$

$$\frac{P_{n+1}}{K} = r \left(1 - \frac{P_n}{K} \right) \frac{P_n}{K}$$

$$x_{n+1} = r(1 - x_n) x_n$$

Logistic map

$$L(x_n) = r(1 - x_n) x_n$$

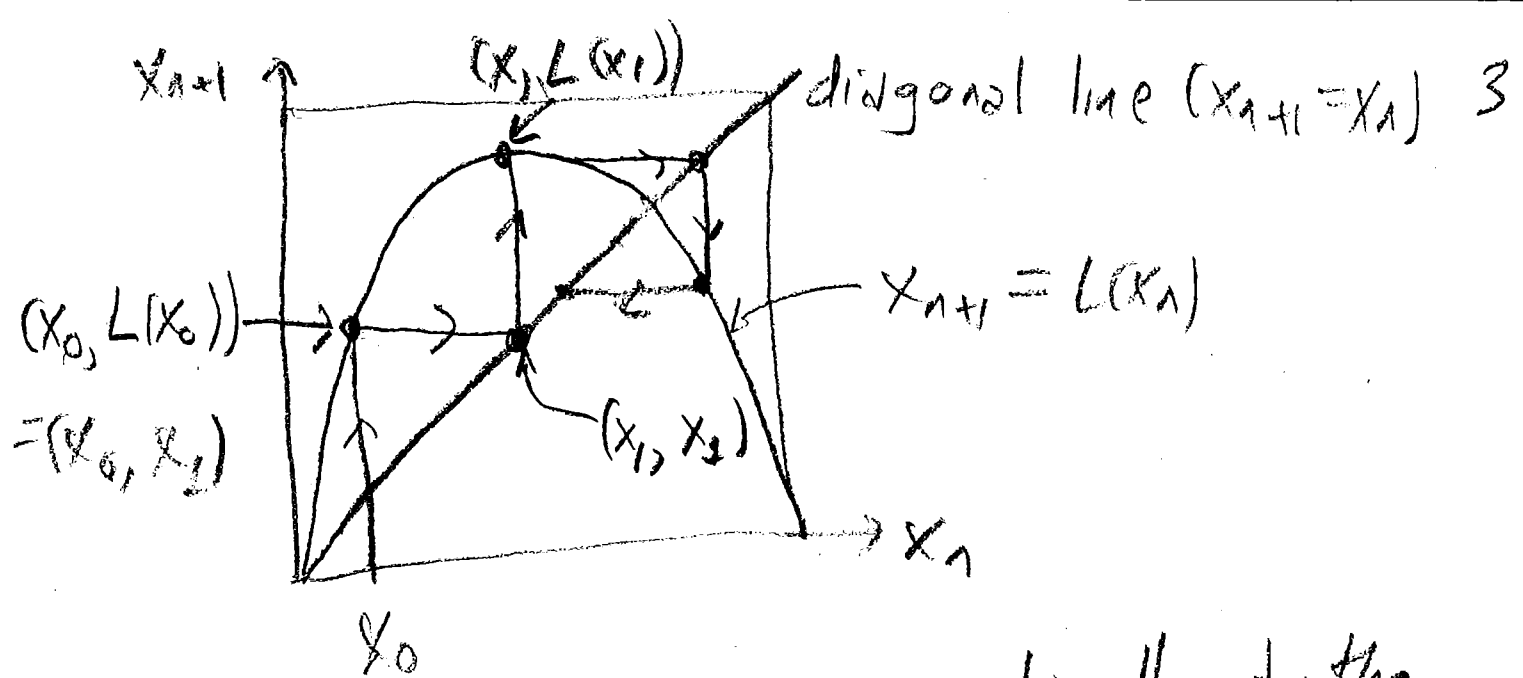


Given x_0 , produce $x_1 = L(x_0)$

$$x_2 = L(x_1)$$

$$x_3 = L(x_2)$$

...



graphical iteration: vertically to the
 graph, horizontally to the diagonal
 visualize the iterates of x_0 .