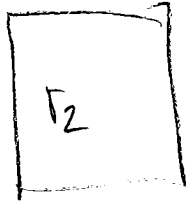
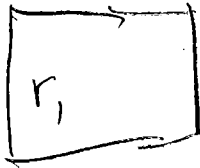


Oct 13 Notes
random fractal construction

①

r_3

r_4



Suppose $r_1 = \begin{cases} 1/2 & \text{with prob} = 1/2 \\ 1/4 & \text{with prob} = 1/2 \end{cases}$

$r_2 = r_3 = \begin{cases} 1/2 & \text{with prob} = 1/3 \\ 1/4 & \text{with prob} = 2/3 \end{cases}$

$r_4 = \begin{cases} 1/2 & \text{with prob} = 1/4 \\ 1/4 & \text{with prob} = 3/4 \end{cases}$

$E(r_1^d + r_2^d + r_3^d + r_4^d) = 1$
"expected value"
"average"

$$E(r_1^d) + E(r_2^d) + E(r_3^d) + E(r_4^d) = 1$$

$$E(r_1^d) = \frac{1}{2} \left(\frac{1}{2}\right)^d + \frac{1}{2} \left(\frac{1}{4}\right)^d$$

$$E(r_2^d) = E(r_3^d) = \frac{1}{3} \left(\frac{1}{2}\right)^d + \frac{2}{3} \left(\frac{1}{4}\right)^d$$

$$E(r_4^d) = \frac{1}{4} \left(\frac{1}{2}\right)^d + \frac{3}{4} \left(\frac{1}{4}\right)^d$$

randomized Moran equation

③

$$\underbrace{\frac{1}{2} \left(\frac{1}{2}\right)^d + \frac{1}{2} \left(\frac{1}{4}\right)^d}_{E(r_1^d)} + \underbrace{\frac{1}{3} \left(\frac{1}{2}\right)^d + \frac{2}{3} \left(\frac{1}{4}\right)^d}_{E(r_2^d)} + \underbrace{\frac{1}{3} \left(\frac{1}{2}\right)^d + \frac{2}{3} \left(\frac{1}{4}\right)^d}_{E(r_3^d)} + \underbrace{\frac{1}{4} \left(\frac{1}{2}\right)^d + \frac{3}{4} \left(\frac{1}{4}\right)^d}_{E(r_4^d)} = 1$$

$$\left(\frac{2}{3} + \frac{3}{4}\right) \cdot \left(\frac{1}{2}\right)^d + \left(\frac{4}{3} + \frac{5}{4}\right) \left(\frac{1}{4}\right)^d = 1 \quad x = \left(\frac{1}{2}\right)^d, x^2 = \left(\frac{1}{4}\right)^d$$

$$\frac{31}{12} x^2 + \frac{17}{12} x - 1 = 0$$