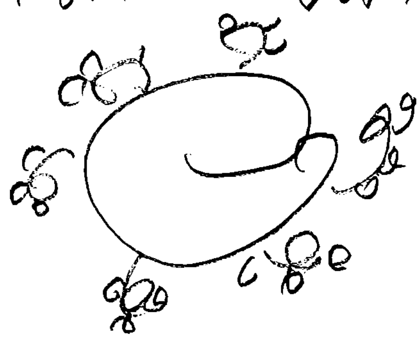
Oct 13 Review

Multifractals

uneven distribution of energy in turbulence from a vortex to its daughters



IFS example

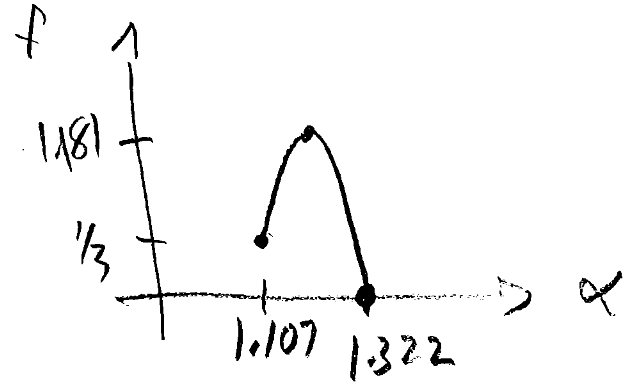
$\text{prob} = r^{\alpha}$

$\alpha = \dfrac{\log(\text{prob})}{\log(r)}$

r	prob	
$\frac{1}{2}$	.4	$\alpha = \dfrac{\log(.4)}{\log(.5)} = 1.322$
$\frac{1}{4}$	.2	$\alpha = \dfrac{\log(.2)}{\log(.25)} = 1.161$
$\frac{1}{4}$	.2	
$\frac{1}{8}$	.1	$\alpha = \dfrac{\log(.1)}{\log(.125)} = 1.107$
$\frac{1}{8}$	.1	

max $\alpha = 1.322$

min $\alpha = 1.107$



$f(\min \alpha)$ comes from the two $r=\frac{1}{8}$ factors

$$N=2,\ r=1/8$$

$$f(\min \alpha) = \frac{\log(2)}{\log(1/1/8)} = \frac{\log(2)}{\log(8)} = \frac{\log(2)}{\log(2^3)}$$

$$= \frac{\cancel{\log(2)}}{3\cdot\cancel{\log 2}} = \frac{1}{3}$$

$f(\max \alpha)$ comes from the single $r=\frac{1}{2}$ factor

$$N=1,\ r=1/2$$

$$f(\max \alpha) = \frac{\log(1)}{\log(1/1/2)} = 0$$

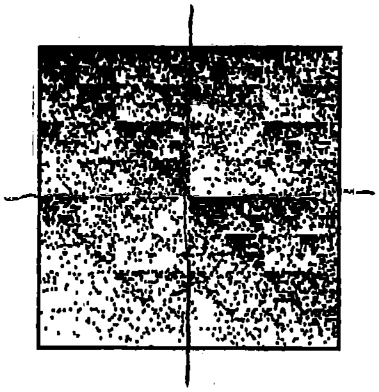
The maximum value on the $f(\alpha)$ is the dimension of the whole IFS

$$\left(\frac{1}{2}\right)^d + 2\left(\frac{1}{4}\right)^d + 2\left(\frac{1}{8}\right)^d = 1$$

$$x = \left(\frac{1}{2}\right)^d \qquad x + 2x^2 + 2x^3 = 1$$

numerically, $x = .441$

$$d = \frac{\log(x)}{\log(1/2)} = \frac{\log(.441)}{\log(.5)} = 1.181$$



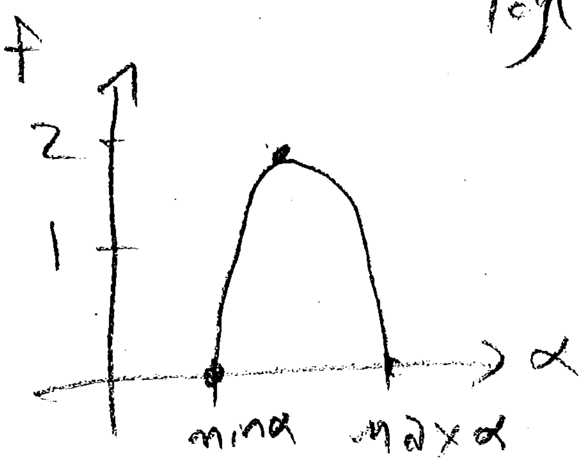
1 is the lightest
3 is the darkest
2 and 4 are middle filled in

min α ~~occurs on~~ corresponds to the max prob (because all scaling factors are 1/2), at corner 3 - apply only T_3

$$f(\min\alpha) = \frac{\log(1)}{\log(1/1/2)} = 0$$

max α corresponds to min prob, at corner 1, apply only T_1

$$f(\max\alpha) = \frac{\log(1)}{\log(1/1/2)} = 0$$



The highest point on the f(α) curve is the dimension of the IFS ($N=4$, $r=1/2$)

$$= \frac{\log(4)}{\log(1/1/2)} = \frac{\log(4)}{\log(2)} = 2$$