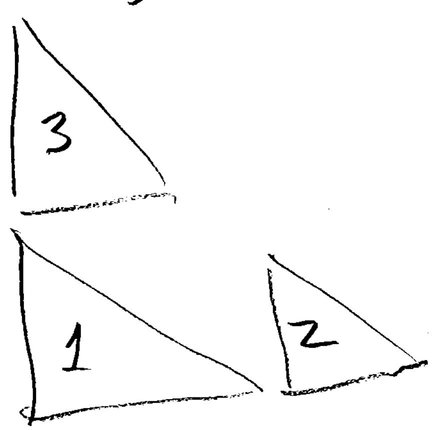
Review

Random fractals
- randomized IFS
- Brownian motion and its friends

randomized IFS allow scaling factor to change



Scaling factors r_1, r_2 and r_3 can vary.

randomized Moran eq.

$$E(r_1^d) + E(r_2^d) + E(r_3^d) = 1$$

Example:

$$r_1 = \begin{cases} 1/2 & \text{with prob} = 1/2 \\ 1/4 & \text{with prob} = 1/2 \end{cases} \qquad r_2 = \begin{cases} 1/2 & \text{with prob} = 1/4 \\ 1/4 & \text{with prob} = 3/4 \end{cases}$$

$$r_3 = \begin{cases} 1/2 & \text{with prob} = 3/4 \\ 1/4 & \text{with prob} = 1/4 \end{cases}$$

Then $r_1^d = \begin{cases} (1/2)^d & \text{with prob } 1/2 \\ (1/4)^d & \text{with prob } 1/2 \end{cases}$ $\quad E(r_1^d) = \frac{1}{2}\left(\frac{1}{2}\right)^d + \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)^d$

$$\underbrace{\frac{1}{2}\left(\frac{1}{2}\right)^d + \frac{1}{2}\left(\frac{1}{4}\right)^d}_{E(r_1{}^d)} + \underbrace{\frac{1}{4}\left(\frac{1}{2}\right)^d + \frac{3}{4}\left(\frac{1}{4}\right)^d}_{E(r_2{}^d)} + \underbrace{\frac{3}{4}\left(\frac{1}{2}\right)^d + \frac{1}{4}\left(\frac{1}{4}\right)^d}_{E(r_3{}^d)} = 1$$

$$\frac{3}{2}\left(\frac{1}{2}\right)^d + \frac{3}{2}\left(\frac{1}{4}\right)^d = 1 \qquad x = \left(\frac{1}{2}\right)^d$$

Then $x^2 = \left(\frac{1}{4}\right)^d$

$$\frac{3}{2}x + \frac{3}{2}x^2 = 1$$

$$3x + 3x^2 = 2$$

$$3x^2 + 3x - 2 = 0 \qquad a = 3 \quad b = 3 \quad c = -2$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 3 \cdot (-2)}}{2 \cdot 3}$$

$$= \frac{-3 \pm \sqrt{33}}{6}$$

Take $x = \dfrac{-3 + \sqrt{33}}{6}$

$$\left(\frac{1}{2}\right)^d = x = \frac{-3 + \sqrt{33}}{6}$$

$$\operatorname{Log}\left(\left(\frac{1}{2}\right)^d\right) = \operatorname{Log}\left(\frac{-3 + \sqrt{33}}{6}\right)$$

$$d = \frac{\operatorname{Log}\left(\frac{-3 + \sqrt{33}}{6}\right)}{\operatorname{Log}(1/2)} \approx 1.128$$

Brownian motion

- The increments (jumps) exhibit a scaling
  waiting K times as long to measure the
  jumps increases the variation of the
  jumps by $\sqrt{K} = K^{1/2}$
- The jumps are independent of one another
- The jumps are distributed according
  to the bell curve - large jumps are
  very rare

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fractional Brownian motion

- The jumps depend on one another
- The jumps are distributed according
  to the bell curve

Levy flights

- The jumps are independent of one another
- The jumps follow a power law, not
  a bell curve