Oct 6 Notes

.3	.3
.1	.3

.03	.03	.09	.09
.01	.03	.03	.09

etc.

Hölder:

prob of being in a square with address $\underbrace{ij \ldots k}_{n \text{ digits}} = \Big(\underbrace{\tfrac{1}{2^n}}_{\text{side length of the square}}\Big)^{\alpha}$

α is the Hölder exponent

Smallest probability:

length 1 $\quad .1$

length 2 $\quad .01 = .1^2$

length 3 $\quad .001 = .1^3$

length n $\quad .1^n$

Highest probability of length n is $.3^n$

~~smallest prob~~ $\cancel{\alpha = \log(.1^n)}$

$$\boxed{\text{prob} = \left(\tfrac{1}{2n}\right)^{\alpha}}$$

$$\text{Log}(\text{prob}) = \log\left(\left(\tfrac{1}{2n}\right)^{\alpha}\right)$$

$$\text{Log}(\text{prob}) = \alpha \log\left(\tfrac{1}{2n}\right)$$

$$\alpha = \frac{\text{Log}(\text{prob})}{\text{Log}\left(\tfrac{1}{2n}\right)}$$

smallest prob. $\alpha = \dfrac{\log(.1^n)}{\log(.5^n)} = \dfrac{\cancel{n}\log(.1)}{\cancel{n}\log(.5)}$

$$= \frac{\log(.1)}{\log(.5)} \approx 3.322$$

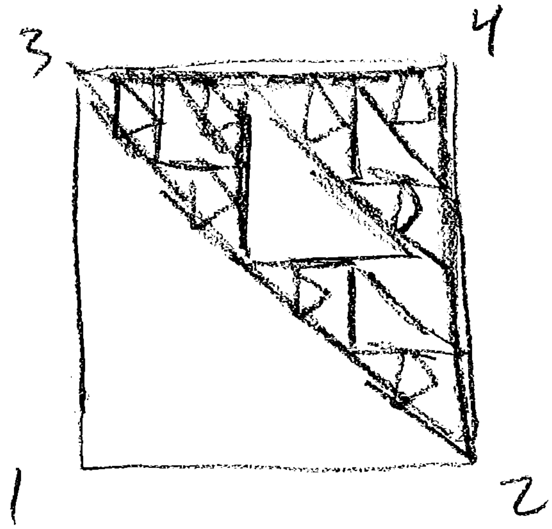
largest prob $\alpha = \dfrac{\log(.3^n)}{\log(.5^n)} = \dfrac{\cancel{n}\log(.3)}{\cancel{n}\log(.5)}$

$$= \frac{\log(.3)}{\log(.5)} \approx 1.737$$

If all the scaling factors are the same $= r$

$$\max \alpha = \frac{\log(\text{min prob})}{\log(r)}$$

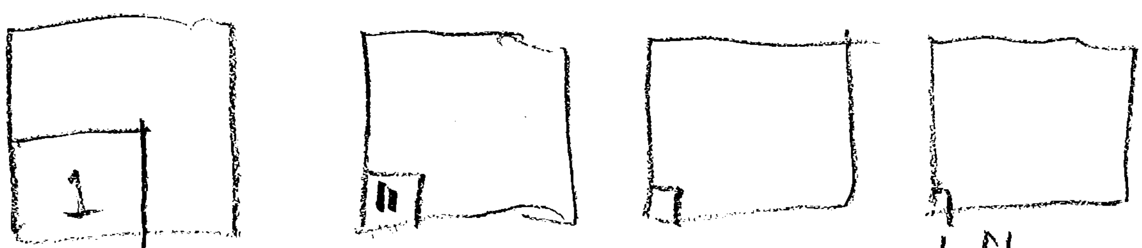
$$\min \alpha = \frac{\log(\text{max prob})}{\log(r)}$$



apply T_2, T_3, T_4

234 gasket has the highest prob, so the lowest α

Apply T_1 over and over

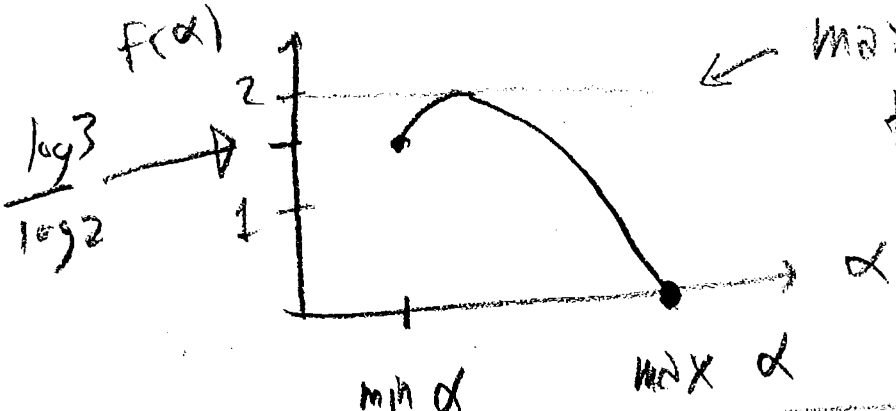


Only applying T_1 gives the lower left corner point.

$\dim(\min\alpha) = \dim(\text{gasket}) = \dfrac{\log 3}{\log 2}$

$\dim(\max\alpha) = \dim(\text{lower left corner}) = 0$

For every α, $f(\alpha)$ denotes the dimension of the set where that α occurs



$\max(f(\alpha)) = \dim$ of the IFS: $N=4$, $r=1/2$

$\dfrac{\log 4}{\log 2} = 2$

Suppose $p_1 = p_2 = .1$ and $p_3 = p_4 = .4$

Find $\min\alpha$, $\max\alpha$, $f(\min\alpha)$, $f(\max\alpha)$