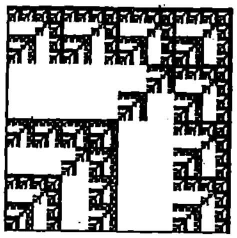
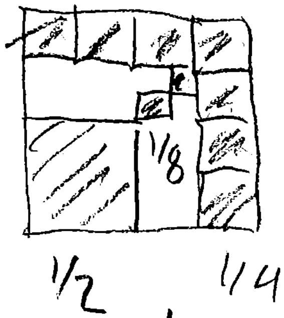
Oct 6 · Review





Moran equation

$$\left(\tfrac{1}{2}\right)^d + 7\left(\tfrac{1}{4}\right)^d + 2\left(\tfrac{1}{8}\right)^d = 1$$

Write $x = (1/2)^d$. Then $(1/4)^d = x^2$ and $(1/8)^d = x^3$

Moran becomes

$$x + 7x^2 + 2x^3 = 1$$

or $$2x^3 + 7x^2 + x - 1 = 0$$

This is a cubic, so try dividing

$$
\begin{array}{r|l}
 & 2x^2 + 5x - 4 \\
\hline
x+1 & 2x^3 + 7x^2 + x - 1 \\
 & -\;\underline{2x^3 + 2x^2} \\
 & \quad 5x^2 + x \\
 & \quad -\;\underline{5x^2 + 5x} \\
 & \qquad -4x - 1 \\
 & \qquad -\;\underline{-4x - 4} \\
 & \qquad\qquad 3
\end{array}
$$

remainder, so $x+1$ is not a factor of $2x^3 + 7x^2 + x - 1$

$$\begin{array}{r}
x^2 + 3x - 1 \\
\text{Try. } 2x+1 \overline{\big)\, 2x^3 + 7x^2 + x - 1} \\
-\underline{\left(2x^3 + x^2\right)} \\
6x^2 + x \\
-\underline{\left(6x^2 + 3x\right)} \\
-2x - 1 \\
-\underline{\left(-2x - 1\right)} \\
0
\end{array}$$

It worked!!!

That is

$$2x^3 + 7x^2 + x - 1 = 0 \quad \text{(Moran)}$$

is the same as

$$(2x+1)(x^2+3x-1) = 0$$

Either $2x+1=0$, so $x = -1/2$ Impossible because $x = (1/2)^d$

Or $x^2 + 3x - 1 = 0$

$a=1 \quad b=3 \quad c=-1$

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot (-1)}}{2 \cdot 1}$$

$$= \frac{-3 \pm \sqrt{13}}{2} \qquad \text{Take } x = \frac{-3+\sqrt{13}}{2}$$

$$\left(\tfrac{1}{2}\right)^d = x, \text{ so } d = \frac{\log(x)}{\log(1/2)} = \frac{\log\left(\frac{-3+\sqrt{13}}{2}\right)}{\log(1/2)}$$

Algebra of Dimensions

1. The dimension of a part cannot exceed the dimension of the whole.
(Monotonicity)

2. If we transform a space by rotation, reflection, translation, or scaling, we do not change its dimension.
(Invariance)

3. $d(A \times B) = d(A) + d(B)$
A and B lie in perpendicular spaces
(product rule)

4. $d(A \cup B) = \max\{d(A), d(B)\}$
(union rule) A and B lie in the same space

5. If A and B lie in n-dimensional space, typically $d(A \cap B) = d(A) + d(B) - n$
If $d(A \cap B) < 0$, then A and B don't intersect.

Example: Find the smallest dimension (whole number) of a space so that two gaskets in that space typically don't intersect.

Don't intersect means $d($$\cap$$) < 0$

$d($$\cap$$) = d($$) + d($$) - n$

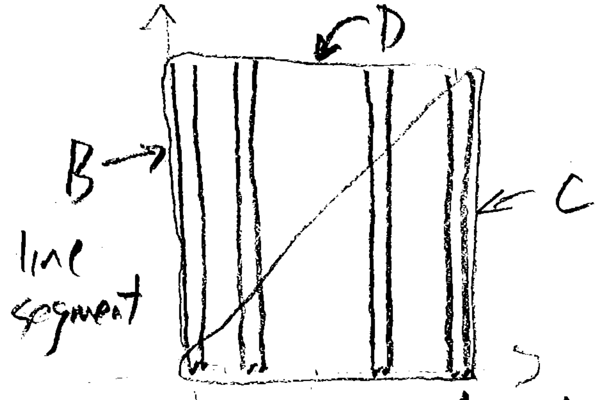
$$= \frac{\log 3}{\log 2} + \frac{\log 3}{\log 2} - n < 0$$

so $$2 \cdot \frac{\log 3}{\log 2} < n$$

$$2 \cdot 1.58 < n$$

$$3.16 < n$$

So $n = 4$



A = Cantor MTS

$$d(A \times B) = d(A) + d(B)$$
$$= \frac{\log 2}{\log 3} + 1$$

Take C to be the half of $A \times B$ below the diagonal line. What is $d(C)$?

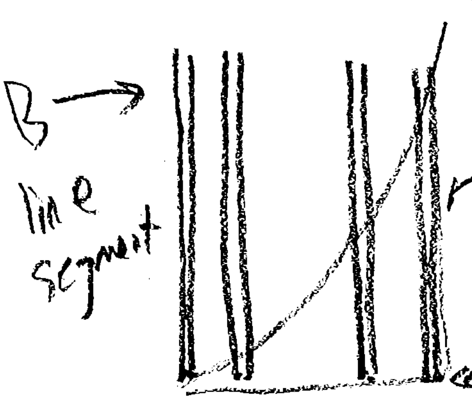
Take D to be the other half of $A \times B$

$$C \cup D = A \times B$$
$$d(C \cup D) = d(A \times B) = \frac{\log 2}{\log 3} + 1$$
$$\parallel$$
$$\max\{d(C), d(D)\}$$

D is C rotated, so $d(C) = d(D)$

so $\max\{d(C), d(D)\} = d(C)$, and $d(C) = \frac{\log 2}{\log 3} + 1$

$y = x^2$



C is the part of $A \times B$ under the parabola. Find $d(C)$.

A = Cantor MTS