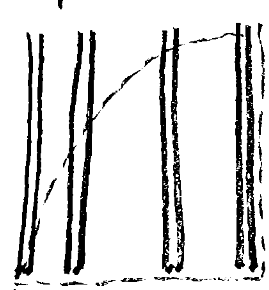
Review

"contained in"
"is a subset of"

Properties of Dimension

1. If A is a part of B ($A \subseteq B$), then $d(A) \le d(B)$
2. Dimension is unchanged by rotation, reflection, translation, or scaling
3. $d(A \times B) = d(A) + d(B)$
   A and B lie in perpendicular spaces
4. $d(A \cup B)$ is the larger of $d(A)$, $d(B)$
   A and B lie in the same space
5. If A and B lie in n-dimensional space, then typically
   $$d(A \cap B) = d(A) + d(B) - n$$

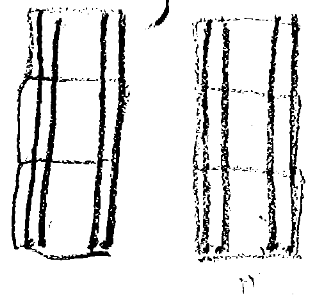
Problem: In the product of a Cantor MTS and a line segment



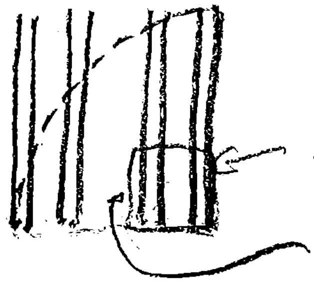
$$d(C \times L) = d(C) + d(L) = \frac{\log 2}{\log 3} + 1$$

A is the portion of $C \times L$ contained in a quarter-circle.
Because A is part of $C \times L$, $d(A) \le d(C \times L)$

To recognize $C \times L$ as self-similar,



Each of these 6 pieces has the same dimension as the whole



A

small copy of $C \times L$ a part of A

$$\frac{\log 2}{\log 3} + 1 = d(\text{small copy of } C \times L) \le d(A) \le d(C \times L) = \frac{\log 2}{\log 3} + 1$$

So $d(A) = \dfrac{\log 2}{\log 3} + 1$

Multifractals

3	4
1	2

p_1 = prob. of applying $T_1 = .1$

$p_2 = .2$

$p_3 = p_4 = .35$

Is there a power law (scaling) relating the probability of visiting an address to the side length of the square with that address?

prob of being in a box $=$ (box size)$^{\alpha}$

$$\log(\text{prob}) = \log((\text{box size})^{\alpha}) = \alpha \cdot \log(\text{box size})$$

$$\alpha = \frac{\log(\text{prob})}{\log(\text{box size})}$$

length n address box has size $\frac{1}{2^n}$

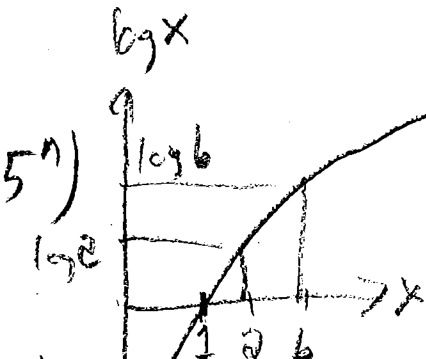
$p_1 = .1$, $p_2 = .2$, $p_3 = p_4 = .35$

all length n probabilities

min prob

$$.1^n \le \text{prob} \le .35^n$$

$$\log(.1^n) \le \log(\text{prob}) \le \log(.35^n)$$



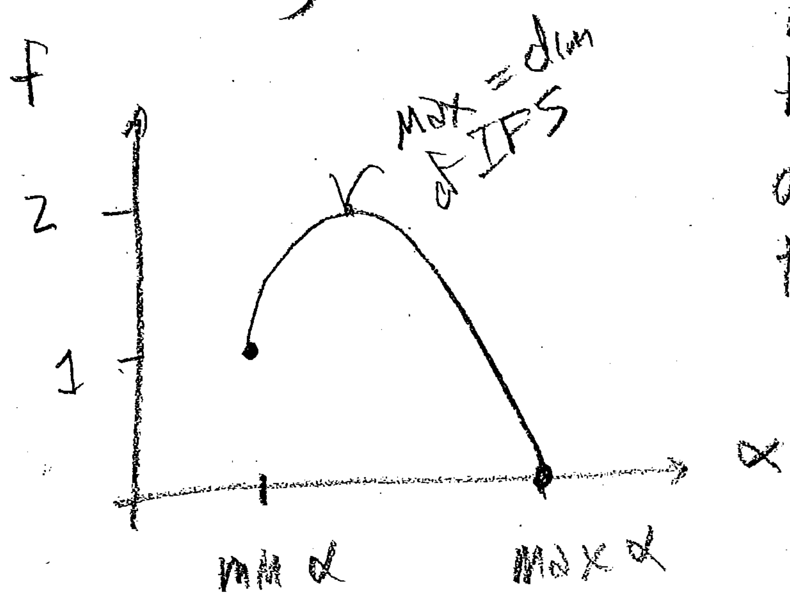
log is negative for numbers < 1

$$\frac{\log(.1^n)}{\log(1/2^n)} \geqslant \frac{\log(\text{prob})}{\log(1/2^n)} \geqslant \frac{\log(.35^n)}{\log(1/2^n)}$$

$$\underbrace{\frac{\cancel{n}\log(.1)}{\cancel{n}\log(.5)}}_{=\,\max\alpha} \geqslant \alpha \geqslant \underbrace{\frac{\cancel{n}\log(.35)}{\cancel{n}\log(.5)}}_{=\,\min\alpha}$$

If all the scaling factors (r) are the same, max prob $\Leftrightarrow$ min α
min prob $\Leftrightarrow$ max α

If some r_i are different, compute each $\frac{\log(p_i)}{\log(r_i)}$.



For each α, $f(\alpha)$ is the dim. of the space where that α occurs.

min α $\Leftrightarrow$ max prob $= .35$, which occurs for T_3 and T_4

$$f(\text{min } \alpha) = \dim\{T_3, T_4\} \quad N=2$$

$$= \frac{\log(2)}{\log(1/(1/2))} \quad r = 1/2$$

$$\boxed{= \frac{\log 2}{\log(2)} = 1}$$

max α $\Leftrightarrow$ min prob $= .1$
which occurs for T_1

$$f(\text{max } \alpha) = \dim\{T_1\}$$

$$N=1, \; r = 1/2$$

$$= \frac{\log(1)}{\log(1/(1/2))} = \frac{\log(1)}{\log(2)} = 0$$