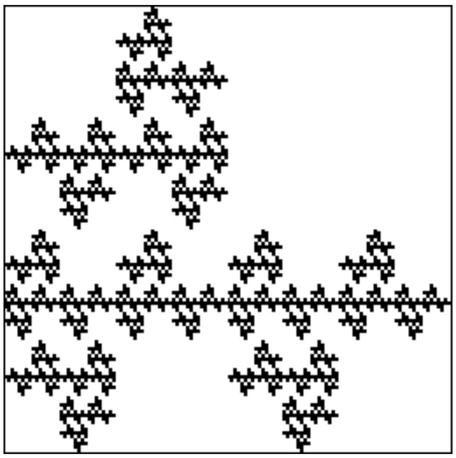
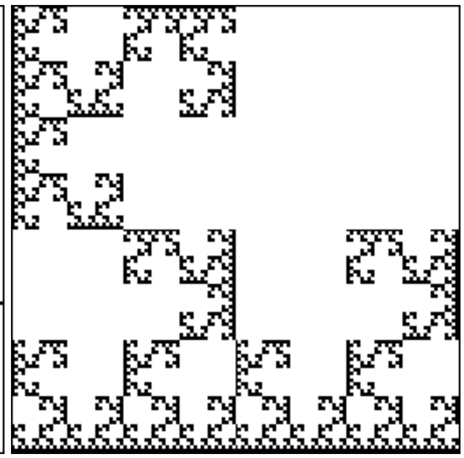
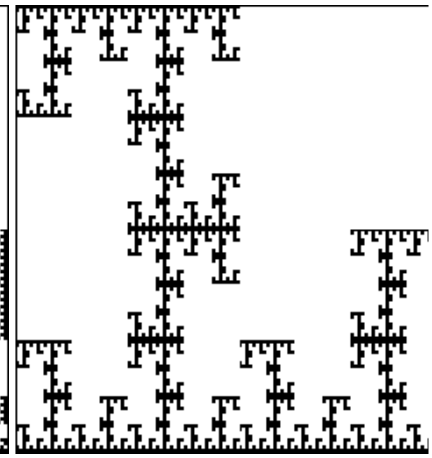
Practice Final 1

This practice final is meant to suggest the level and number of problems on the real final. Topics not covered on the practice final can appear on the real final; topics covered on the practice final need not appear on the real final. The best use of this practice final is to find three hours you can work without interruption. Take the final and compare your answers with the solutions.

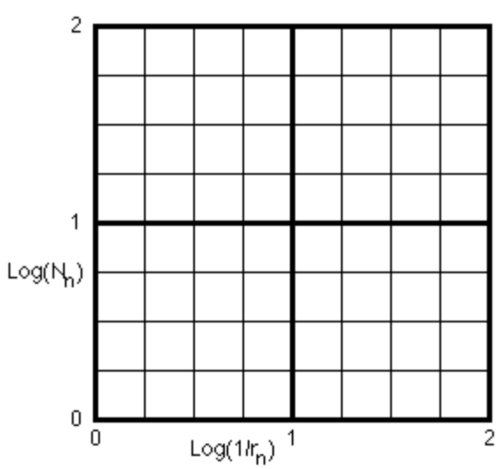
1. Find IFS rules for these fractals.

2. Suppose covering a fractal with boxes gives these values of N_n, the number of boxes, and r_n, the side length of the boxes.

r_n	N_n	$Log(1/r_n)$	$Log(N_n)$
0.25	3	0.60	0.48
0.125	5	0.90	0.70
0.0625	8	1.20	0.90
0.03125	13	1.51	1.11

(a) Plot the appropriate points on the graph and find the dimension.



(b) Suppose the points of the graph were shifted vertically by 0.25. How would the dimension of the fractal change? Explain your answer.

(c) Based on your answer to (a), do you think this fractal can contain any straight lines? (Note fractals can contain straight lines: think of the Sierpinski gasket.)

3. Define a function B(x) by

B(x) = 2x for x <= 1/2
B(x) = 2x - 1 for x > 1/2

So for example, B(2/5) = 4/5 and B(4/5) = 8/5 - 1 = 3/5. Starting from a number x_0, determine the numbers $x_1 = B(x_0)$, $x_2 = B(x_1)$, $x_3 = B(x_2)$, $x_4 = B(x_3)$, and $x_5 = B(x_1)$, (be careful with the arithmetic) for

(a) $x_0 = 1/3$,

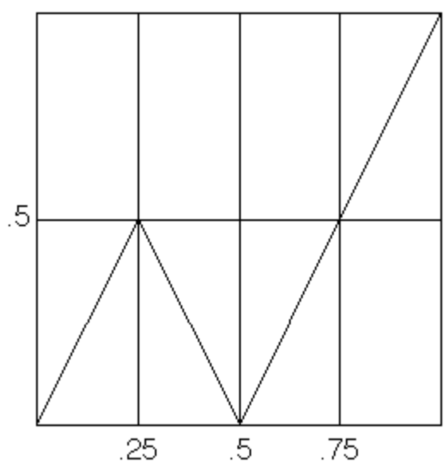
(b) $x_0 = 1/7$, and

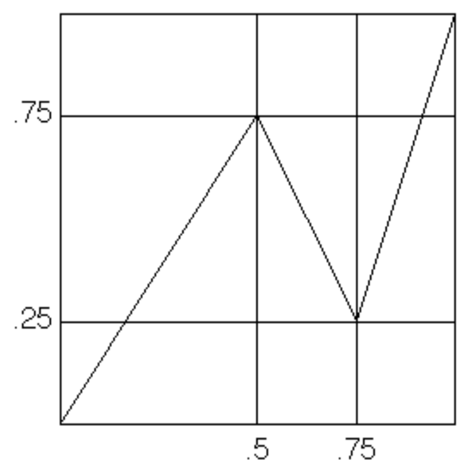
(c)$x_0 = 1/15$.

(d) In each case, what cycle do you obtain?

(e) Noting that $3 = 2^2 - 1$, $7 = 2^3 - 1$, and $15 = 2^4 - 1$, support the statement “No matter how near 0 we look, we will find points belonging to some cycle for B(x).”

4. Pictured here are generators for two random processes.

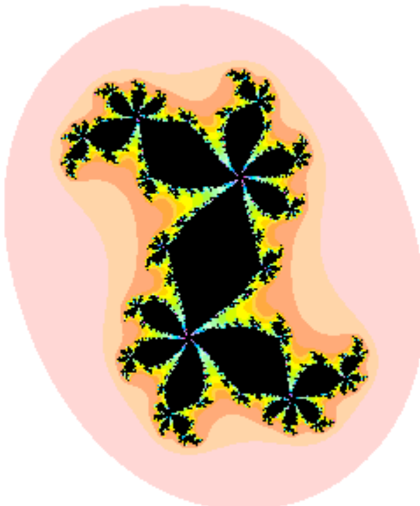


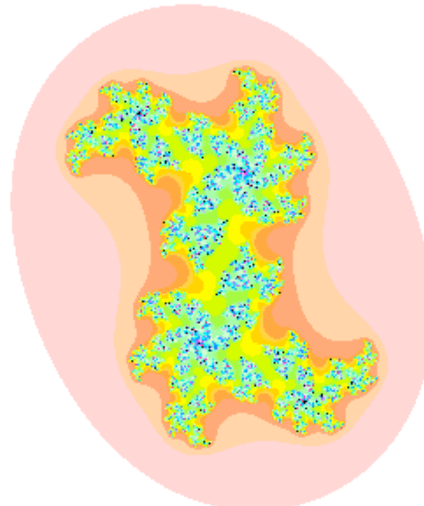


(a) Are either of these unifractal? Give a reason to support your answer.

(b) Will either of these produce Brownian motion? Give a reason to support your answer.

5. Pictured below are Julia sets for two values of c.

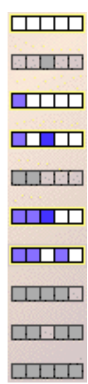




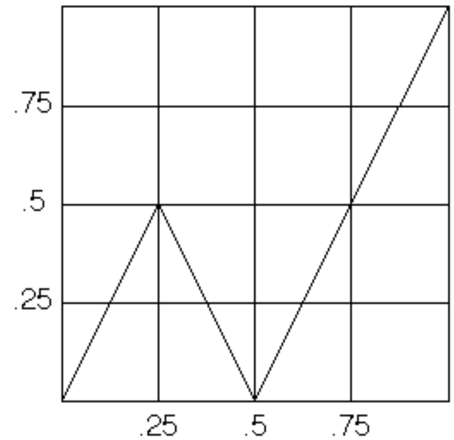
(a) Does either c belong to the Mandelbrot set? Give a reason to support your answer for both pictures.

(b) For each c in part (a) that belongs to the Mandelbrot set, give the length of the cycle to which the iterates of $z_0 = 0$ converges. Give a reason to support your answer.

6. For the CA with this rule, suppose the first generation consists of all dead cells. Describe completely all future generations. Give a reason to support your answer.



7. The graph of the function pictured here is used to drive an IFS (using the standard transformations).



(a) List the empty length 2 addresses in the driven IFS.

(b) On the left picture, shade those empty length 2 addresses.

(c) On the right picture, shade the empty length 3 addresses. Explain how you got your answer. (Hint: is this a Markov partition? Why or why not?)