# Practice Final Solutions

1.

| R | S | Theta | Phi | E | F |
|---|---|---|---|---|---|
| 0.5 | -0.5 | 0 | 0 | 0 | 0.5 |
| 0.5 | -0.5 | 0 | 0 | 0.5 | 0.5 |
| -0.5 | 0.5 | 0 | 0 | 0.5 | 0.5 |

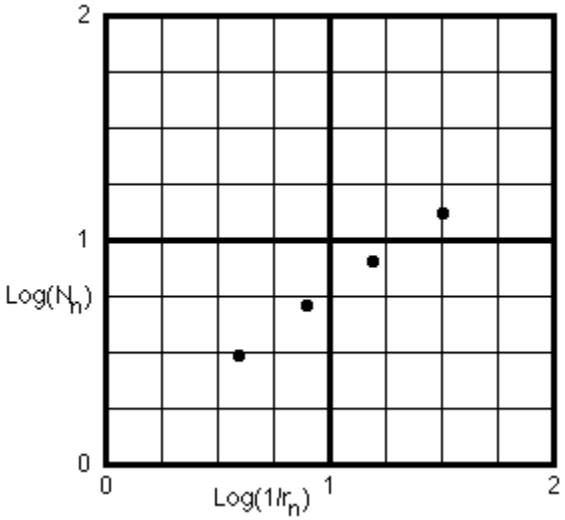
| R | S | Theta | Phi | E | F |
|---|---|---|---|---|---|
| -0.5 | 0.5 | 0 | 0 | 0.5 | 0 |
| -0.5 | 0.5 | 0 | 0 | 1 | 0 |
| 0.5 | 0.5 | -90 | -90 | 0 | 1 |

| R | S | Theta | Phi | E | F |
|---|---|---|---|---|---|
| -0.5 | 0.5 | 0 | 0 | 0.5 | 0 |
| -0.5 | 0.5 | 0 | 0 | 1.0 | 0 |
| -0.5 | -0.5 | 0 | 0 | 0.5 | 1 |

2. (a) The dimension is the slope of the line determined by these points. The points all appear to lie along a line, so to find the slope compute rise over run using the first and last points.

(1.11 - .48)/(1.51 - .6) = .63/.91 = .69

You won't be using calculators, so it is good enough to estimate this as .63/.9 = .7. Yes, I do expect you to be able to perform simple arithmetic without a calculator.




(b) Shifting the graph vertically does not change the slope of the line, so the dimension is unaltered. That is, the fractal still would have dimension about .7.

(c) A straight line has dimension 1, this fractal has dimension .7. A lower-dimensional object cannot contain a higher-dimensional object, so this fractal cannot contain a straight line. (You might recall the exercise where we saw that the dimension of a shape made up of the gasket and a line had dimension that of the gasket. Usually, the dimension of an object made of several pieces is the largest dimension of any of the pieces. So if the fractal in this problem contained a straight line, its dimension would have to be at least 1. You don't need to go into this much detail in your answer, just recall a lower-dimensional object cannot contain a higher-dimensional object.)

3.

| | $x_0$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ |
|---|---|---|---|---|---|---|
| (a) | 1/3 | 2/3 | 4/3 - 1 = 1/3 | 2/3 | 4/3 - 1 = 1/3 | 2/3 |
| (b) | 1/7 | 2/7 | 4/7 | 8/7 - 1 = 1/7 | 2/7 | 4/7 |
| (c) | 1/15 | 2/15 | 4/15 | 8/15 | 16/15 - 1 = 1/15 | 2/15 |

(d) In (a) we have a 2-cycle because two numbers, 1/3 and 2/3, repeat. In (b) we have a 3-cycle because three numbers, 1/7, 2/7, and 4/7, repeat. In (c) we have a 4-cycle because four numbers, 1/15, 2/15, 4/15, and 8/15, repeat.

(e) The pattern appears to be $1/(2^n - 1)$ belongs to an n-cycle. For larger and larger n these fractions get closer and closer to 0, so points belonging to cycles for B(x) are as close to 0 as we like.

4. (a) The left picture generator has four pieces, each with dY = 0.5 and dt = 0.25. Consequently, for each piece of the generator we have Log(dY)/Log(dt) = Log(0.5)/Log(0.25). Because we have the same ratio for each piece, the left generator is unifractal.

For the right generator, the second and third pieces both have dt = 0.25, yet the second has dY = 0.5 and the third has dY = 0.75. Without computing the values, we can tell the right generator is not unifractal because the second and third pieces have different dY but the same dt.

(b) The left will produce Brownian motion because for all pieces of the generator we have the Brownian scaling $dt = dY^2$: $0.25 = 0.5^2$ and $0.25 = (-0.5)^2$. Because Brownian motion is unifractal and the right generator is not unifractal, the right generator will not give Brownian motion.

5. (a) The left Julia set is connected, so its c-value belongs to the Mandelbrot set. The right Julia set is a Cantor set, so its c-value does not belong to the Mandelbrot set.

(b) For the left Julia set, the iterates of $z_0 = 0$ converge to a 5-cycle. We can see this because at every branch point of the Juila set, five pieces of the Julia set meet.

6. If the first generation consists of all dead cells, the second generation will have all cells alive because the *all dead* configuration is selected. Selecting this configuration means that a dead cell surrounded by four dead cells will become alive in the next generation. Every cell in the first generation is a dead cell surrounded by four dead cells, so every cell will become alive in the second generation.

In the third generation, all cells become dead. This is because the configuration *all alive* is not selected. Not selecting this configuration means that a live cell with four live neighbors becomes dead in the next generation. The second generation consists of all live cells, so every cell in the second generation is a live cell surrounded by four live cells. Consequently, every cell in the second generation will die, and the third generation will consist of all dead cells.

The third generation is the same as the first, so the fourth generation will be the same as the second. Continuing this pattern, all odd generations will be completely dead, all even generations will be

completely alive.

7. (a) The allowed transitions are

1 -> 1,2; 2 -> 1,2; 3 -> 1,2; 4 -> 3,4

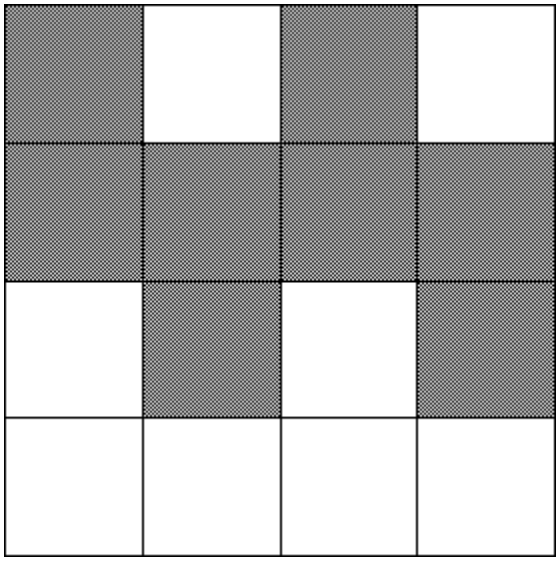
so the forbidden transitions are

1 -> 3,4; 2 -> 3,4; 3 -> 3,4; 4 -> 1,2

and the empty length 2 addresses are

31, 41, 32, 42, 33, 43, 14, 24

(b) Here are the empty length 2 addresses shaded.



(c) This is a Markov partition, because in the graph of the function

bin 1 goes to all of bins 1 and 2

bin 2 goes to all of bins 1 and 2

bin 3 goes to all of bins 1 and 2

bin 4 goes to all of bins 3 and 4

Consequently, every empty length 3 address must contain an empty length 2 address. Here is the picture with the empty length 3 addresses shaded.