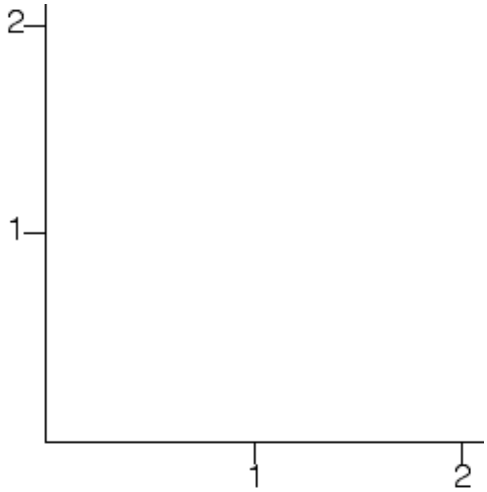
Practice Final 2

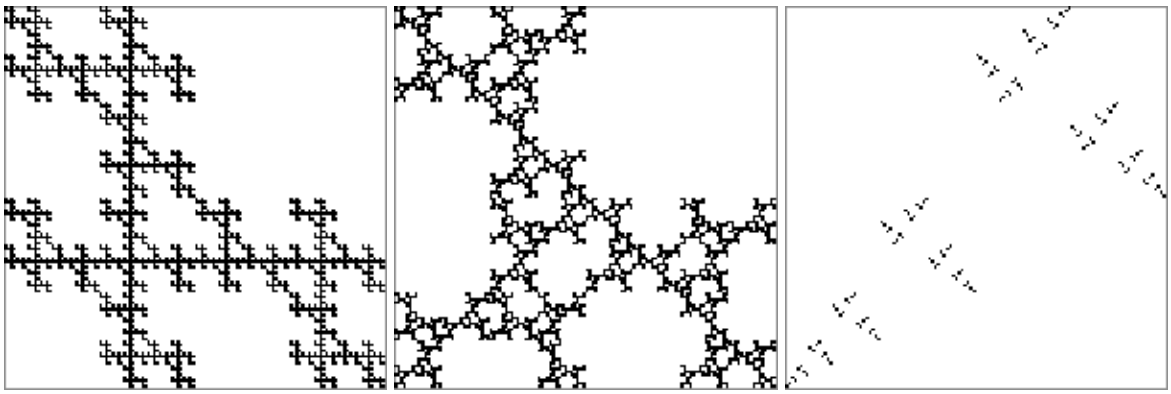
This practice final is meant to suggest the level and number of problems on the real final. Topics not covered on the practice final can appear on the real final; topics covered on the practice final need not appear on the real final. The best use of this practice final is to find three hours you can work without interruption. Take the final and compare your answers with the solutions.

1. (a) On this graph, sketch points the could come from measuring the mass dimension of a fractal of mass dimension 1.5. Be sure to label the axes. (That is, say what is plotted along each axis.



(b) Explain why you think your points represent an answer to this question.

2. Find IFS rules to generate each of these fractals. Note each fractal is contained in the unit square with the origin at the lower left corner.



3. (a) Suppose a fractal shape in the plane is made up of pieces each scaled by a factor of 1/4. What is the maximum number of such pieces making up the fractal, provided the pieces do not overlap except along their edges. Give a reason to support your answer.

(b) Consider this IFS of the right isosceles gasket

r	s	theta	phi	e	f
0.5	0.5	0	0	0	0

0.5	0.5	0	0	0.5	0
0.5	0.5	0	0	0	0.5

Can changing the translations (e and f) of the rules alter the dimension of the shape? Give a reason to support your answer.

4. (a) Give the formula for generating the Mandelbrot set and the Julia sets. (Either the real or the complex version will do.)

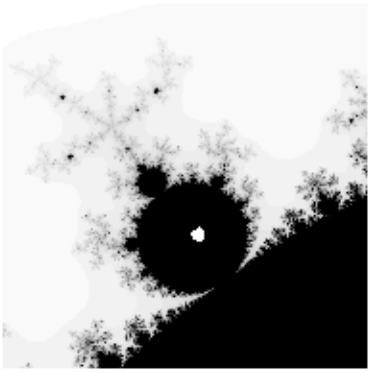
(b) Describe how the formula of 4. (a) is used to generate the Mandelbrot set.

(c) Describe how the formula of 4. (a) is used to generate Julia sets.

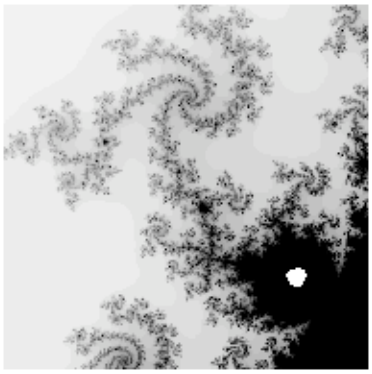
(d) Explain the relation between the Mandelbrot set and the Julia sets. That is, which Julia sets correspond to points in the Mandelbrot set?

5. Here are three regions of the Mandelbrot set, and three Julia sets. Each Julia set corresponds to the small white circle in one of the Mandelbrot set pictures. Say which Julia set corresponds to which Mandelbrot set point, and give a reason supporting your answer.

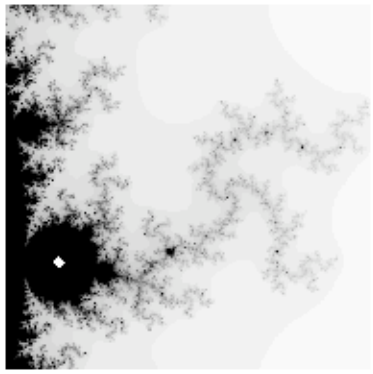
(a)



(b)



(c)



(i)

(ii)

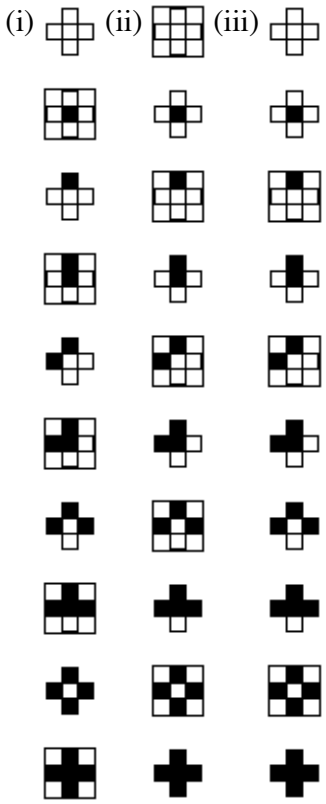
(iii)

6. Pictured here is a time series with the bin boundaries indicated by the horizontal lines.

(a) Give a careful description of the corresponding driven IFS. That is, what shapes will dominate the driven IFS? Include a rough sketch of the driven IFS.

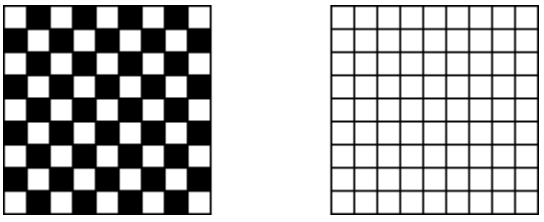
(b) Explain how you reached your conclusion.

7. Here are rules for three von Neumann outer totalistic CA. The configurations enclosed in boxes represent those giving a live cell.

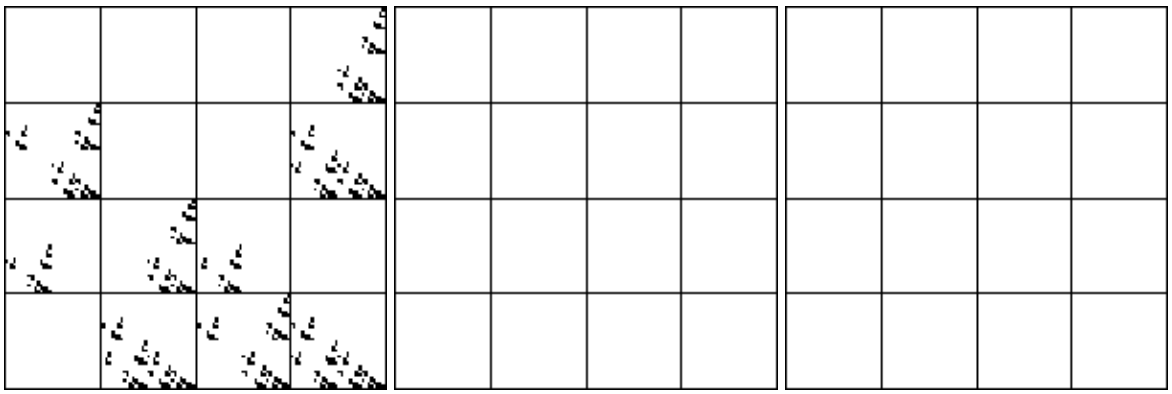


(a) Suppose the first generation is a radnom distribution of live and dead cells. In terms of the first generation, describe all future generations of patterns that evolve under rule (i), and all future generations that evolve under rule (ii).

(b) On the left is part of the first generation, an infinite checkerboard. Describe the second generation evolving under rule (iii), and on the right shade the live cells of this part of the second generation.



8. (a) In the middle draw the graph of a function that will produce the driven IFS on the left.



(b) Explain why your graph will produce this driven IFS. Your answer should mention addresses and Markov partitions.

(c) On the right draw the graph of another function (that is, having a graph different from your answer to (b)) that will produce the driven IFS on the left.

9. (a) Describe the principal ways in which chaos differs from randomness.

(b) Describe the implications of the Trading Time theorem in terms of large events and dependent events.