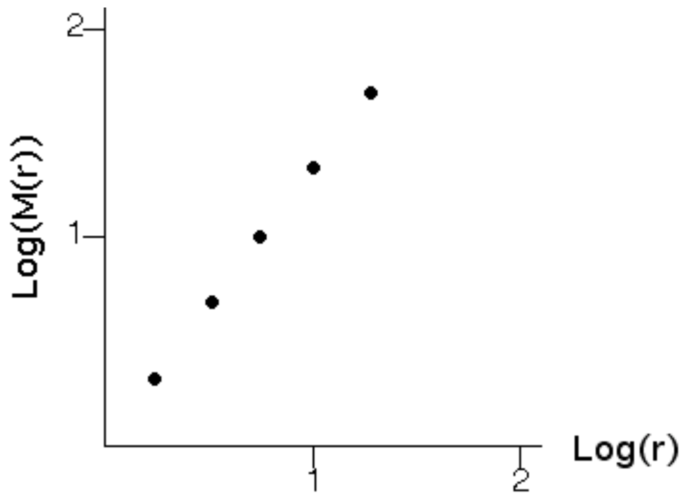
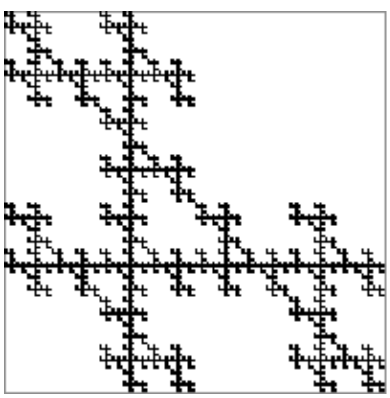
Practice Final 2 Answers

1. (a) Here is a sketch. The horizontal axis is labeled Log(r), the vertical Log(M(r)).

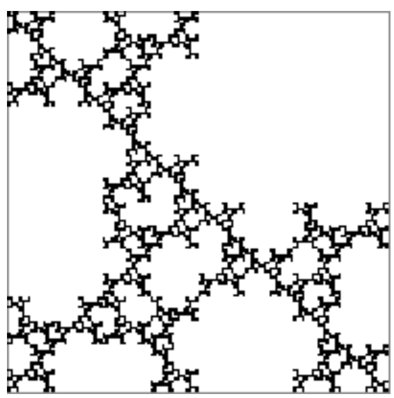


(b) The points lie along a straight line of slope 1.5. Under the scaling hypothesis, $M(r) = kr^d$, the log-log plot reveals points on a line of this slope.

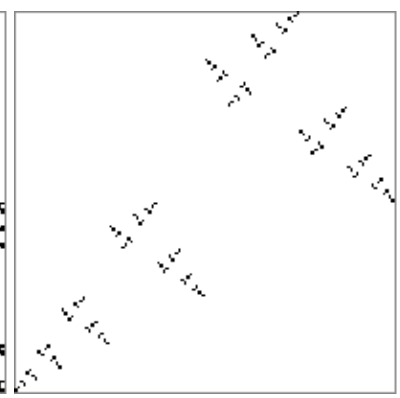
2. Here are the IFS rules for these fractals.



R	S	Theta	Phi	E	F
0.5	0.5	180	180	0.5	0.5
0.5	0.5	180	180	1.0	0.5
0.5	0.5	180	180	0.5	1.0



R	S	Theta	Phi	E	F
0.5	0.5	90	90	0.5	0.0
0.5	0.5	180	180	1.0	0.5
0.5	0.5	180	180	0.5	1.0



R	S	Theta	Phi	E	F
0.5	0.5	0	0	0	0.0
0.5	0.5	90	90	1.0	0.5

3. (a) For a fractal made of N pieces each scaled by 1/4, the dimension is Log(N)/Log(4). Because the dimension of any subset of the plane cannot exceed 2, N cannot exceed 16.

(b) Usually translations cannot change dimension, which depends on only the number and size of the pieces. There is an exception to this observation. Making the translations of the second transformation agree with those of the first reduces the number of transformations to two. In this case the IFS generates a line segment, so this change of translations reduces the dimension from Log(3)/Log(2) to 1.

4. (a) Both the Mandelbrot set and Julia sets are generated by $z_{n+1} = z_n^2 + c$. The real equivalent is $x_{n+1} = x_n^2 - y_n^2 + a$ and $y_{n+1} = 2x_n y_n + b$.

(b) The Mandelbrot set lives in the plane of all c-values. For each c, the iteration begins with $z_0 = 0$. If the iterates z_n diverge to infinity, c does not belong to the Mandelbrot set. Otherwise, c does belong to the Mandelbrot set.

(c) Each c produces a different Julia set. Julia sets live in the plane of z_0 values, and consist of those z_0 for which the iterates z_n do not diverge to infinity.

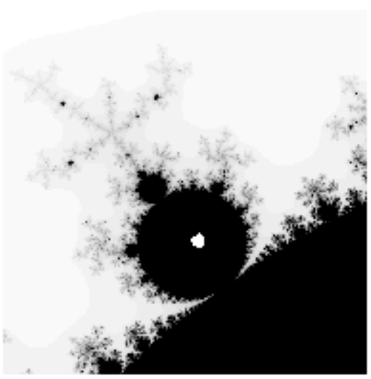
(d) Julia sets are either connected of Cantor sets. The Mandelbrot set is all those c-values for which the Julia set is connected.

5. Counting antenna branches, (a) corresponds to a 5-cycle; counting lobes that join at a common point, (iii) corresponds to a 5-cycle.

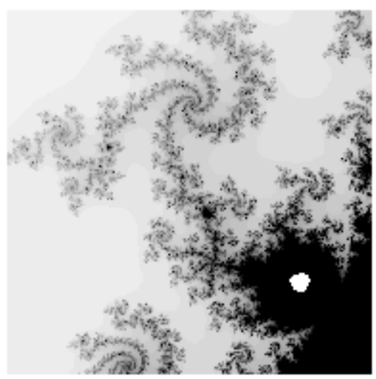
Counting antenna branches, (b) corresponds to a 4-cycle; counting lobes that join at a common point, (ii) corresponds to a 4-cycle.

Counting antenna branches for (c) is more subtle. The most obvious pattern consists of 3 branches, but at least one more pattern appears to be present. Consequently, this disc is attached to a 3-cycle disc. The main feature of the corresponding Julia set is 3 lobes meeting at a point, though other patterns must be present as well. Consequently, (c) corresponds to (i).

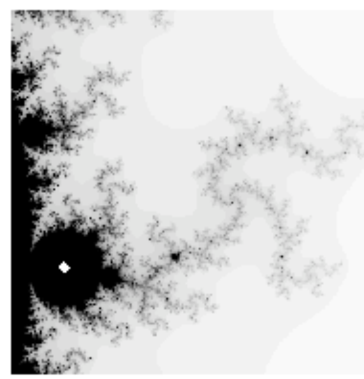
(a)



(b)



(c)



(i)

(ii)

(iii)

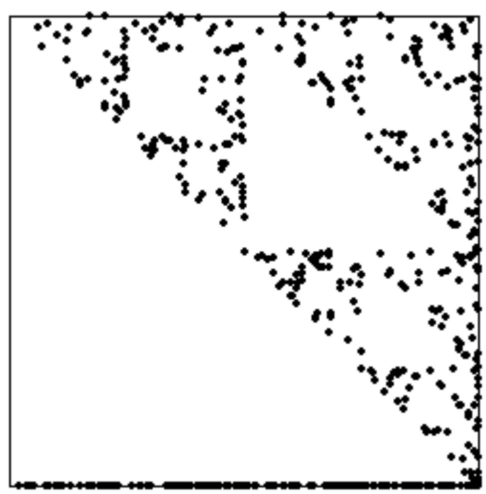
6. Pictured here is a time series with the bin boundaries indicated by the horizontal lines.

(a) and (b) The driven IFS will consist of three main parts, corresponding to the three obvious divisions of the time series.

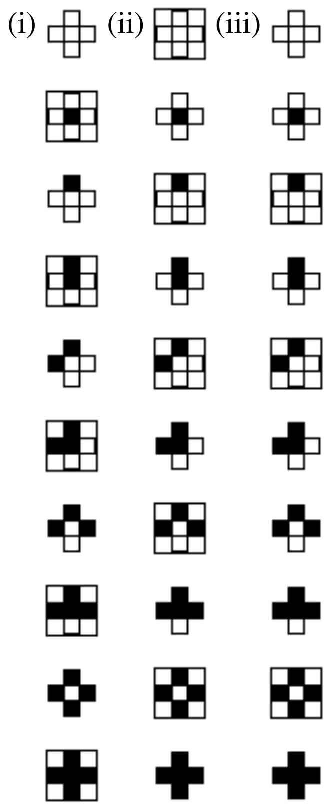
First the data points lie entirely in bins 2, 3, and 4. The driven IFS is contained in the gasket with corners 2, 3, and 4.

In the next region all the data points lie in bin 2, so the driven IFS points head to corner 2.

In the last region all the data points lie in bins 1 and 2, so the driven IFS points lie in the line segment between corners 1 and 2.



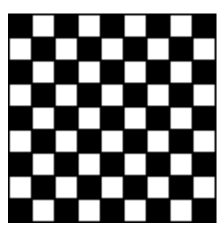
7. Here are rules for three von Neumann outer totalistic CA. The configurations enclosed in boxes represent those giving a live cell.



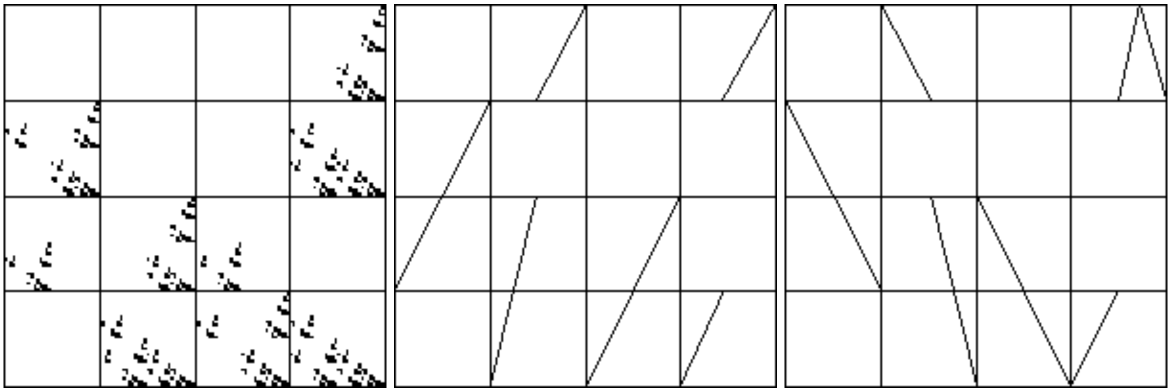
(a) Examining the nbhd configs giving a live cell for CA (i), we see that live cells stay alive (every nbhd config with the central cell alive gives a live cell) and dead cells stay dead (every nbhd config with the central cell dead gives a dead cell). So this CA preserves exactly every initial configuration.

By contrast, for CA (ii) every nbhd config with the central cell dead gives a live cell, and every nbhd config with the central cell alive gives a dead cell. Consequently, this CA exactly reverses the initial configuration, making every dead cell alive and every live cell dead. This alternation repeats for all generations.

(b) Under rule (iii) a dead cell surrounded by four live cells becomes alive, and a live cell surrounded by four dead cells (remember, this is a von Neumann nbhd) dies. So the second generation will be another infinite checkerboard, but with the positions of the live and dead cells interchanged from those of the initial configuration.



8. (a) The middle graph would produce the driven IFS on the left.



(b) The driven IFS has these ovvupied addresses:

12, 13, 14, 21, 22, 23, 31, 42, 44

Consequently, these allowed transitions

1->2, 1->3, 2->1, 2->2, 2->4, 3->1, 3->2, 4->1, 4->4

If the allowed pairs give all the information about the driven IFS, we draw a graph with a Markov partition. Specifically, every bin the graph enters must be crossed completely bottom to top.

(c) On the right the graph of another function crossing the same bins completley. Consequently, this graph will produce the same driven IFS.

9. (a) Choas is deterministic: the past is completely determined by the future. Randomness is not deterministic: the past has no relation to the future. For random systems, even short-term prediction is impossible, except when different outcomes have different probabilities - then those probabilities can be mentioned. Foe chaotic systems, short-term prediction is possible, but not long-term because of sensitivity to initial conditions. Finally, chaotic systems are filled with unstable cycles, while random systems have no such cycles.

(b) Multifractal data have both large events and dependent events. Using the Trading TIme theorem, time can be rescaled to absorb the large events in the changed time. Time is stretched around the large events, making them not so large. This leaves only the signature of the dependent events.