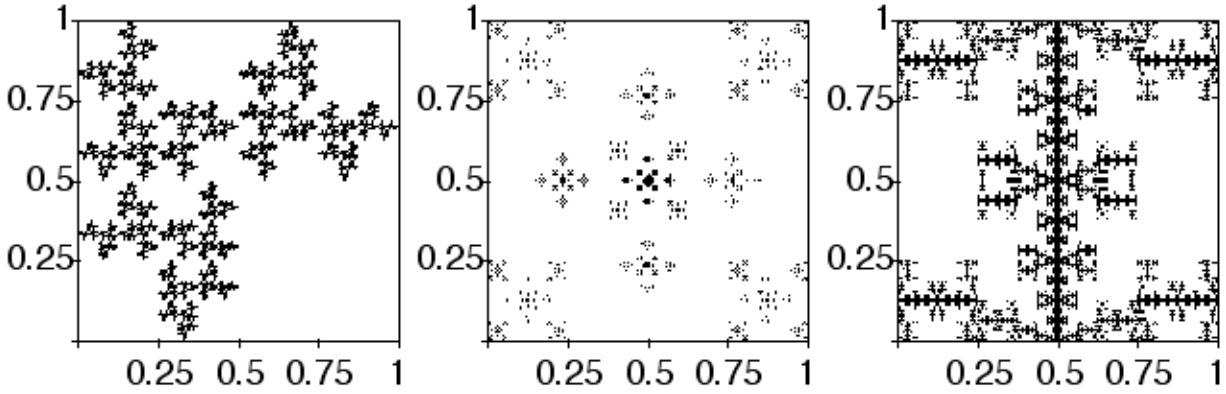
Practice Final 3

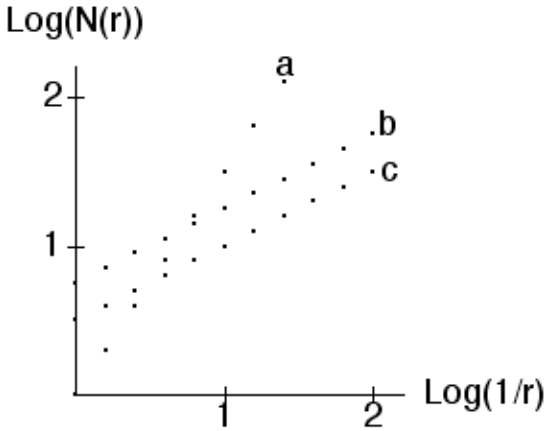
This practice final is meant to suggest the level and number of problems on the real final. Topics not covered on the practice final can appear on the real final; topics covered on the practice final need not appear on the real final. The best use of this practice final is to find three hours you can work without interruption. Take the final and compare your answers with the solutions.

1. Find IFS rules to generate each of these fractals.



2. Compute the dimensions of each of the fractals in problem 1. If the Moran equation is needed, solve the equation using the quadratic formula, not numerically.

3. The three log-log plots, a, b, or c, correspond to the box-counting dimension of three fractals. Which fractlas have the highest dimension, which the lowest? Explain how you arrived at your answer.

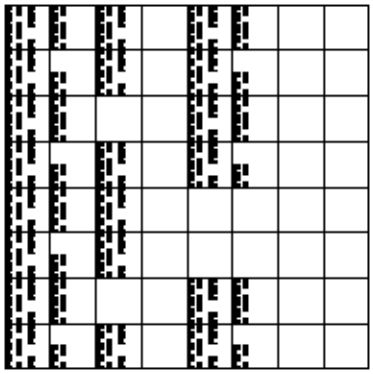


4. (a) What is the typical dimension of the intersection of two Cantor middle-thirds sets in the plane? In 3-dimensional space. Explain how you arrived at your answer.

(b) What is the typical dimension of the intersection of two Sierpinski gaskets in the plane? In 3-dimensional space. In 4-dimensional space. Explain how you arrived at your answer.

(c) Suppose a fractal A has dimension d. Find the smallest (integer) dimension E of the space in which A can be placed so that typically two copies of A will not intersect. Explain how you arrived at your answer.

5. (a) Show the IFS image below is generated by one-step memory (forbidden pairs). Give a detailed explanation of how you arrived at your conclusion. For reference, the length 3 address squares are shown in the drawing.

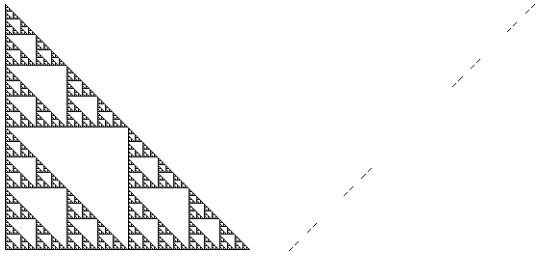


(b) Draw the transition graph that generates this IFS with memory.

(c) Show the IFS image has dimension log(3)/log(2).

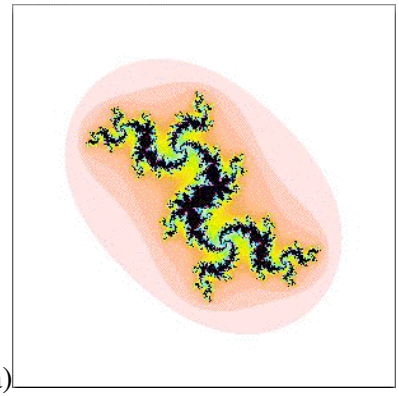
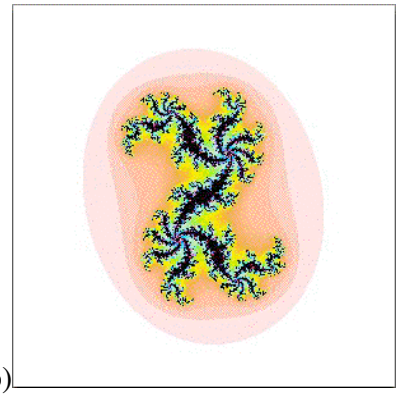
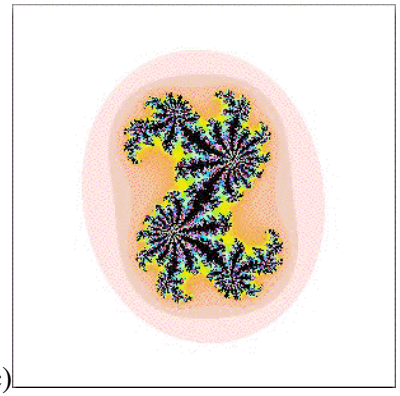
6. Pictured on the left is the set on which the minimum value of α occurs, on the right the set on which the maximum value of α occurs. Suppose the attractor has dimension 2.

(a) Sketch the $f(\alpha)$ curve of this multifractal. Label the axes and indicate the scale of the graph. Label the important points on the graph of the $f(\alpha)$ curve. Explain how you arrived at your answer.

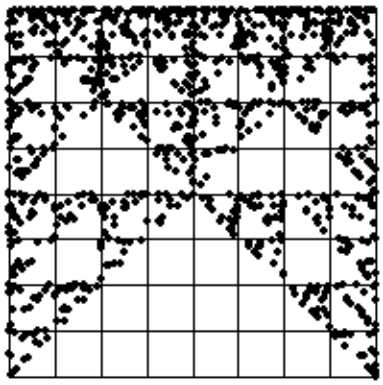


(b) Can this multifractal be generated by an IFS with four transformations? Explain how you arrived at your answer.

7. Which, if any, of these is the Julia set for a c-value for which there is an attracting 15-cycle. Explain how you arrived at your answer.

(a)  (b)  (c) 

8. (a) Pictured below is a driven IFS. For reference the length 3 addresses are included. Using the bin boundaries indicated below the driven IFS picture, sketch a time series that could generate this driven IFS. Explain how you arrived at your answer.



(b) Is this driven IFS generated by forbidden pairs? If so, draw the transition graph. If not, show why not.

9. Pictured below is a price-clock time generator. Find the exact values of the Trading Time generator.