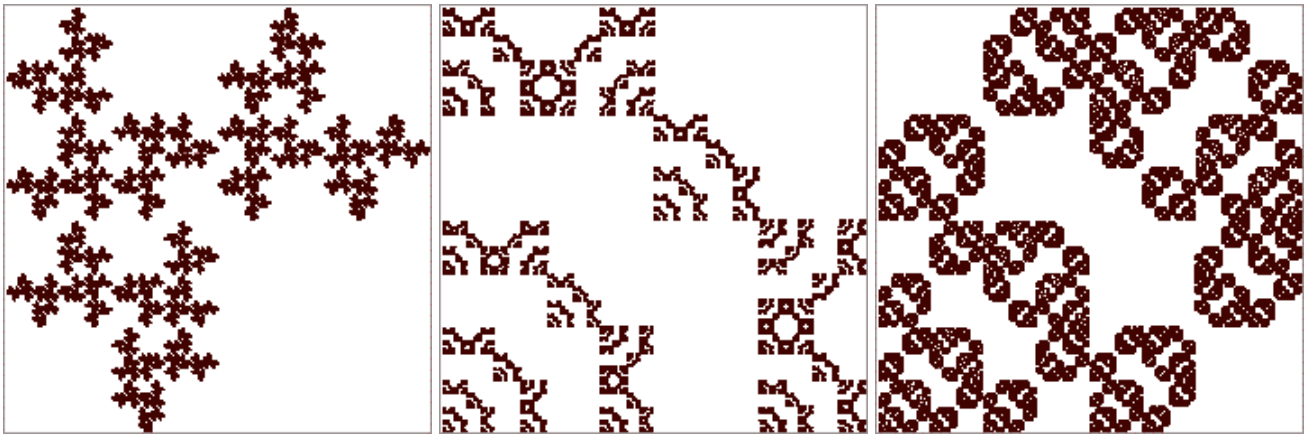
Practice Final 4

This practice final is meant to suggest the level and number of problems on the real final. Topics not covered on the practice final can appear on the real final; topics covered on the practice final need not appear on the real final. The best use of this practice final is to find three hours you can work without interruption. Take the final and compare your answers with the solutions.

1. Find IFS rules to generate each of these fractals.



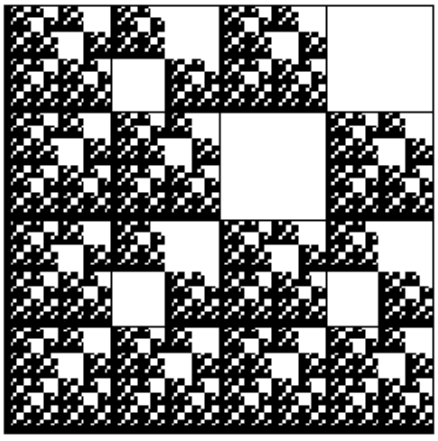
2. Compute the dimensions of each of the fractals in problem 1. If the Moran equation is needed, solve the equation using the quadratic formula, not numerically.

3. Suppose for each n> 0, the minimum number of boxes of side length $\epsilon = 1/2^n$ needed to cover a fractal A is

$$N(\epsilon) = 2^n + 3^n + n$$

Compute the box-counting dimension of A.

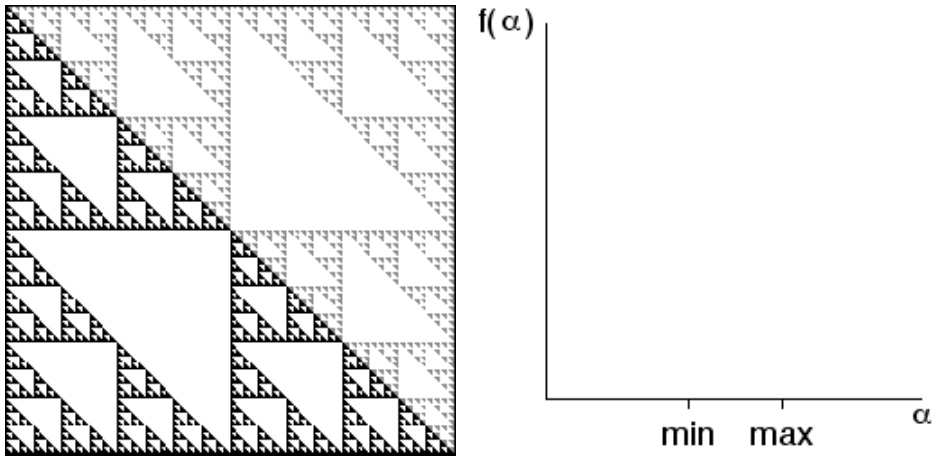
4. (a) Draw the transition graph that generates the IFS with memory.



(b) Can this fractal be generated by an IFS without memory? Support your answer. If it can be, find the IFS rules.

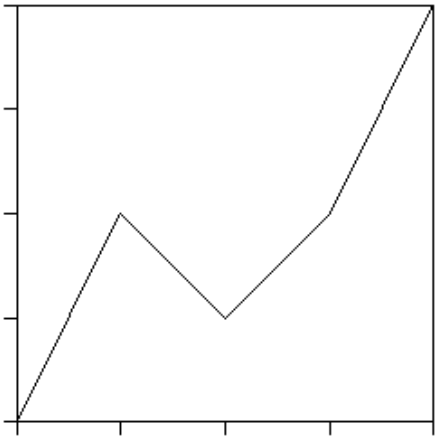
(c) Find the dimension of this fractal.

5. (a) On the left below we see the set of points (dark) with minimum α and the set of points (light) with maximum α value. Assuming the full attractor is the filled-in square, sketch the $f(\alpha)$ curve in the graph on the right. On the vertical axis, label the $f(\alpha)$ values for the minimum and maximum α, and for the maximum point on the $f(\alpha)$ curve.



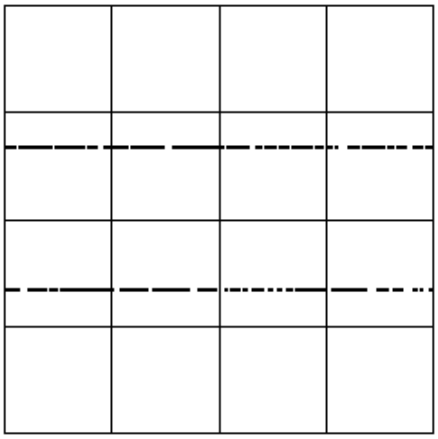
(b) Can this multifractal be generated by the standard four function IFS, $T_1(x,y) = (x/2,y/2)$, $T_2(x,y) = (x/2,y/2) + (1/2,0)$, $T_3(x,y) = (x/2,y/2) + (0,1/2)$, and $T_4(x,y) = (x/2,y/2) + (1/2,1/2)$? Support your answer.

6. (a) Pictured here is the generator for one of Mandelbrot's finance cartoons. Is this unifractal or multifractal? Support your answer.



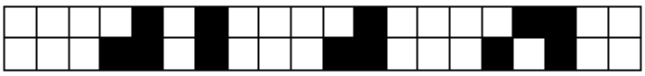
(b) Find the lengths of the trading time generators. You may leave the answers as fractions raised to ugly powers.

7. (a) Pictured below is a driven IFS. The length 2 address squares are shown for reference. Draw the transition graph consistent with this driven IFS.



(b) Sketch a time series that would generate this driven IFS. Explain how your time series will generate this driven IFS.

8. Find all the N = 3 binary CA rules that produce the second row of cells from the first. Support your answer.



9. How many 4-cycle midgets does the Mandelbrot set have? Support your answer. ("I memorized the answer," or "We discussed this in class," are not sufficient reasons.)