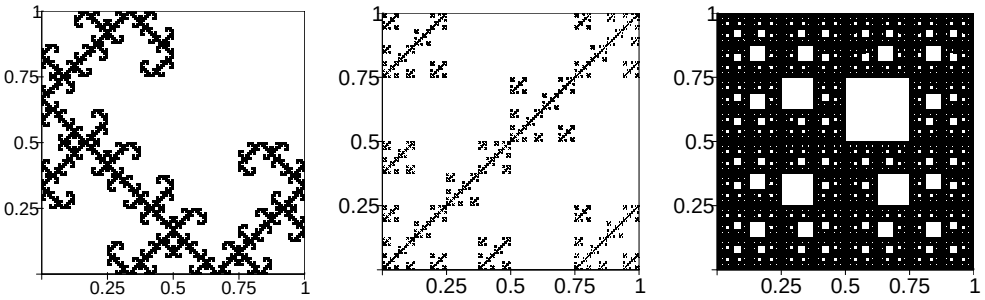
Practice Final Exam 5

1. Find IFS rules to generate these fractals.



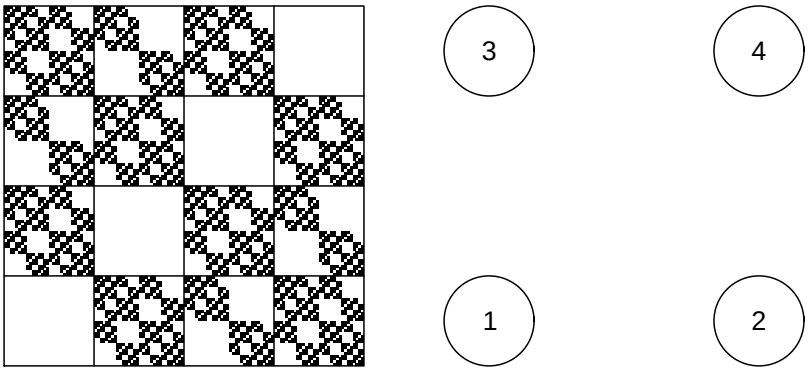
2. Compute the dimensions of each of the fractals in problem 1. If the Moran equation is needed, solve the equation using the quadratic formula, not numerically.

3. Suppose for each $n > 0$, the minimum number of boxes of side length $\epsilon = 1/2^n$ needed to cover a fractal A is

$$N(\epsilon) = n \cdot 3^n$$

Compute the box-counting dimension of A. Hint: $\log(10) = 1$, $\log(10^2) = 2$, $\log(10^3) = 3$, and so on.

4. (a) On the right draw the transition graph that generates the IFS with memory on the left.



(b) Can this fractal be generated by an IFS without memory? Support your answer. If it can be, find the IFS rules.

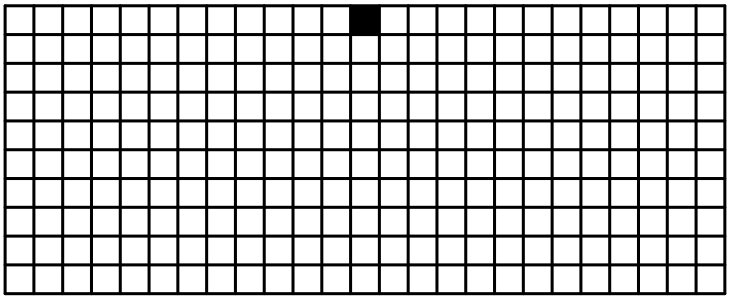
(c) Find the dimension of this fractal.

5. Sketch the $f(\alpha)$ curve generated by this IFS. Label and put scales on the axes. Label the important points, and find the values of $\alpha_{\min}$, $\alpha_{\max}$, $f(\alpha_{\min})$, $f(\alpha_{\max})$, and the maximum value of $f(\alpha)$. Where appropriate, leave your answers as a

ratio of logs; do not give decimal expressions. Give reasons to support your answers.

r	s	θ	φ	e	f	prob
.5	.5	0	0	0	0	0.3
.5	.5	0	0	.5	0	0.3
.25	.25	0	0	0	.5	0.025
.25	.25	0	0	.25	.5	0.025
.25	.25	0	0	0	.75	0.025
.25	.25	0	0	.25	.75	0.025
.5	.5	0	0	.5	.5	0.3

6. Consider the $N = 3$ CA with this rule, (L, D, D), (D, D, L), and (L, D, L) give L, all other nbhds give D. Suppose generation 1 has a single live cell, pictured below. Fill in the rest of the resulting pattern in generations 2 through 10.



How many live cells will be in generation 100? Explain how you arrived at your answer.

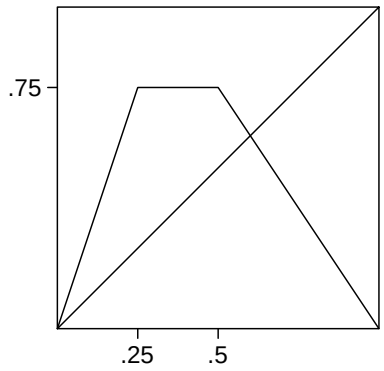
7. This problem refers to the Mandelbrot set. For each of these numbers N, say which of (i), (ii), or (iii) is true. Give a reason to support your choice.
(i) N must be the cycle number of a disc attached to a disc attached to the main cardioid,
(ii) N might be be the cycle number of a disc attached to a disc attached to the main cardioid, or
(iii) N must not be the cycle number of a disc attached to a disc attached to the main cardioid.

(a) $N = 17$

(b) $N = 16$

(c) $N = 15$

8. Pictured here is the graph of a function f inside the unit square, together with the line $y = x$.

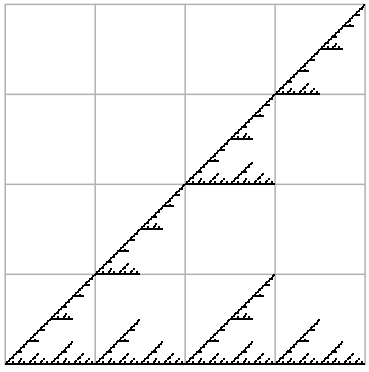


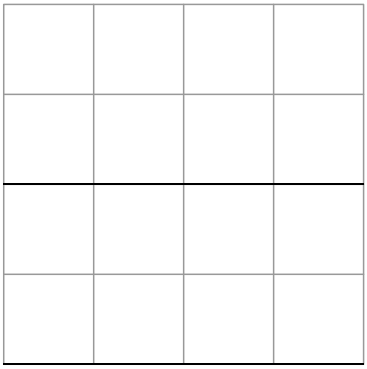
(a) On the graph circle the fixed points of f. Are these fixed points stable or unstable? Give a reason for your answer.

(b) On the graph, plot a 2-cycle for f.

(c) Does the iteration of f exhibit chaos? Explain your answer.

9. (a) Draw the transition graphs of these IFS with memory attractors.

(i) 

(ii) 

9. (b) Sketch a time series that would generate each of these IFS with memory images as driven IFS, or give a reason if you think there is no such driven IFS. Give enough detail to support your claim. Explain how you arrived at your answer.