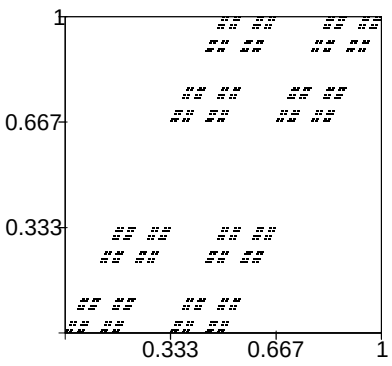
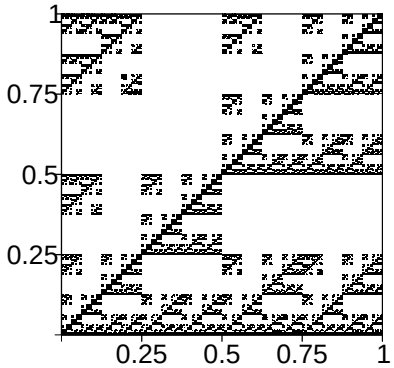
Final Exam

1. Find IFS rules to generate these fractals.



(a)



(b)

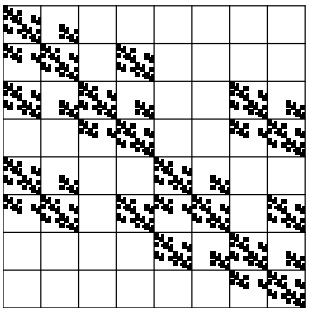
2. Compute the dimensions of each of the fractals in problem 1. If the Moran equation is needed, solve the equation using the quadratic formula, not numerically. Your answer should include logs. Show your calculations, not just your answers.

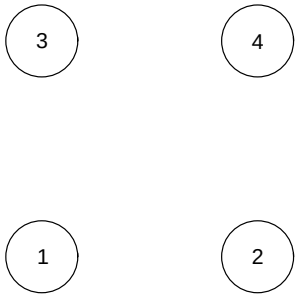
3. Suppose for each $n > 0$, the minimum number of boxes of side length $\epsilon = 1/2^n$ needed to cover a fractal A is

$$N(\epsilon) = 2 \cdot (2^n + 3^n)$$

Compute the box-counting dimension of A. Show your calculations.

4. (a) Show the IFS with memory shown on the left can be generated by forbidden pairs. On the right draw the transition graph that generates this IFS.





4. (b) Can the fractal of 4(a) be generated by an IFS without memory, with a finite number of transformations? Support your answer using properties of the transition graph (romes, etc.). If it can be, find the IFS rules.

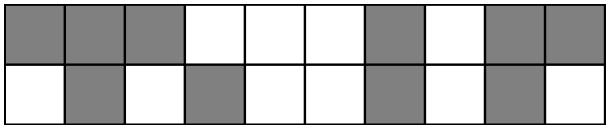
(c) Write the equation to find the dimension of this fractal. You need not solve this equation.

5. Sketch the $f(\alpha)$ curve generated by this IFS. Label and put a scale on the vertical axis. Label the important points, and find the values of $\alpha_{\min}$, $\alpha_{\max}$,

$f(\alpha_{\min})$, $f(\alpha_{\max})$, and the maximum value of $f(\alpha)$. Where appropriate, leave your answers as a ratio of logs; do not give decimal expressions. Give reasons to support your answers.

r	s	θ	φ	e	f	prob
.25	.25	0	0	0	0	0.05
.25	.25	0	0	.25	0	0.05
.25	.25	0	0	.25	.25	0.1
.25	.25	0	0	0	.25	0.1
.25	.25	0	0	.5	.5	0.1
.25	.25	0	0	.75	.75	0.3
.25	.25	0	0	.75	0	0.3

6. Consider a 1-dimensional CA shown here. The top line is the first generation of live and dead cells; the bottom line follows from the first by application of a CA rule.

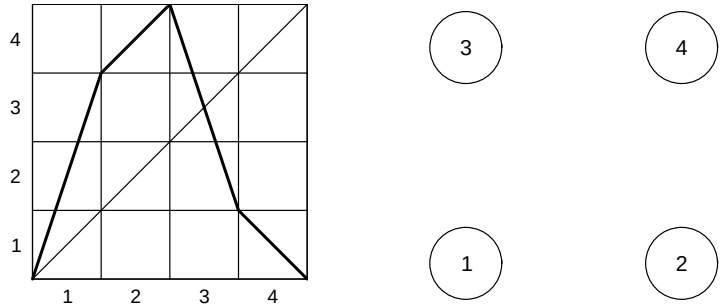


(a) Assuming no wrap-around, write the $N = 3$ CA rule that produces the bottom line from the top. That is, for each of the 8 neighborhood configurations, LLL through DDD, say whether the CA makes the central cell L or D.

(b) Assuming wrap-around, show no $N = 3$ CA could produce the bottom line from the top.

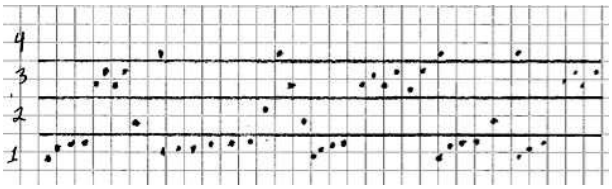
7. Suppose a disc A is attached to a disc B attached to the main cardioid of the Mandelbrot set, and A has cycle number 30. List all the possible cycle numbers of B and explain how you arrived at your answers.

8. The dark lines pictured on the left are the graph of a function f inside the unit square, together with the line $y = x$ and the boundaries of four equal-size bins.



(a) On the right above, draw the transition graph for the driven IFS produced by iterating this function.

(b) Can this be a time series generated by iterating this function? Give a reason to support your answer.



9. The fractal A pictured here is the portion of the product of a Cantor middle thirds set along the x-axis, and the unit interval along the y-axis, inside a circle. Using algebra of dimension results, find the dimension of A. Give the exact answer, not a decimal approximation. Explan how you arrived at your answer.