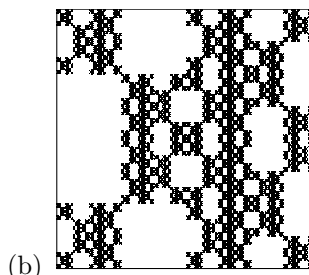
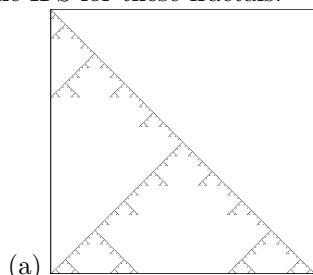


Math 190a Final Exam

1. Find the IFS for these fractals.



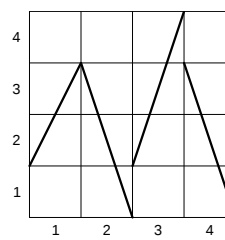
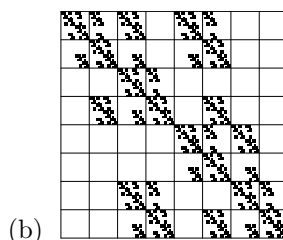
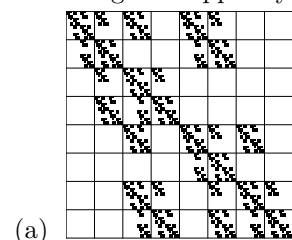
2. Compute the dimensions of both of the fractals in problem 1. Find the exact values, not decimals. Your answer should include logs. Show your calculations, not just your answers.

3. Suppose for each $n > 0$, the minimum number of boxes of side length $\epsilon = 1/4^n$ needed to cover a fractal is

$$N(\epsilon) = 2^n + 3^n + 5$$

Compute the box-counting dimension of this fractal. Show your calculations.

4. Which driven IFS, (a) or (b), is generated by iterating the function graphed on the right? Support your choice.



5. Sketch the $f(\alpha)$ curve generated by this IFS. Put a scale on the vertical axis. Find the values of $\alpha_{\min}$, $\alpha_{\max}$, $f(\alpha_{\min})$, $f(\alpha_{\max})$, and the maximum value of $f(\alpha)$. Leave your answers as a ratio of logs; do not give decimal expressions. Give reasons to support your answers.

r	s	θ	φ	e	f	prob
.25	.25	0	0	0	0	0.2
.25	.25	0	0	.25	0	0.2
.25	.25	0	0	.5	0	0.1
.25	.25	0	0	.75	0	0.1
.25	.25	0	0	.25	.25	0.2
.25	.25	0	0	.5	.5	0.1
.25	.25	0	0	0	.75	0.025
.25	.25	0	0	.25	.75	0.025
.25	.25	0	0	.5	.75	0.025
.25	.25	0	0	.75	.75	0.025

6. Pictured here are the first two generations of a binary CA.

(a) If you think no $N = 3$ CA can produce the second generation from the first, say why. If you think an $N = 3$ CA can produce the second generation from the first, give the rule for that CA.

(b) If there is an $N = 3$ CA producing the second generation from the first, is it the only $N = 3$ CA producing the second generation from the the first. If you think there are no others, say why. If you think there are others, list them.



7. Suppose A is a 5-cycle disc attached to the main cardioid of the Mandelbrot set, and B is a 7-cycle disc attached to the main cardioid of the Mandelbrot set. Are more 35-cycle discs attached to A or to B ? Support your answer.

8. A random Cantor set is constructed by an IFS with two transformations, both scale by $r = 1/2$ with probability $p = 1/2$, and by $r = 1/4$ with probability $p = 1/2$.

(a) Find the expected value of the dimension of this Cantor set. Exact value, please, no decimals.

(b) Find the most likely dimension of the intersection two copies of this Cantor set placed randomly (but nearby) in the plane.

9. This is a transition graph for our standard four-function IFS that generates the filled-in unit square. Find the exact value of the dimension of the fractal determined by this transition graph. Hint: remember that if $|x| < 1$,

$$1 + x + x^2 + x^3 + \cdots = \frac{1}{1 - x}$$

