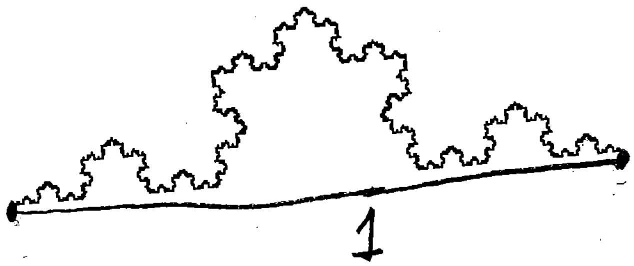
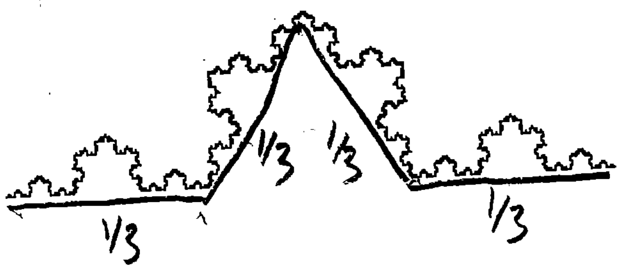
Seat 24 notes

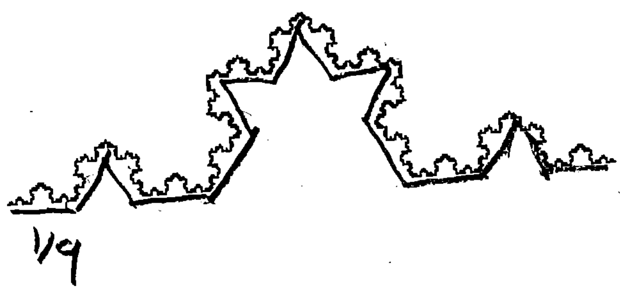
Length of the Koch curve $= L$



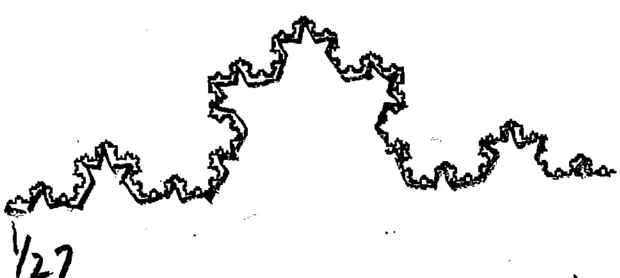
$L > 1$



$L > \frac{4}{3}$



$L > \frac{16}{9} = \left(\frac{4}{3}\right)^2$



$L > \frac{64}{27} = \left(\frac{4}{3}\right)^3$

$L > \left(\frac{4}{3}\right)^n$ for all $n \geq 1$

$L = \infty$

area of the triangle is T

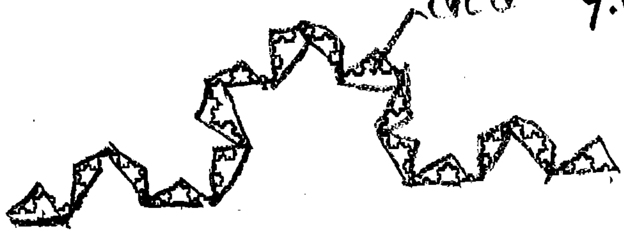
Area of the Koch curve $= A$

$A < T$

area $= \frac{T}{3 \cdot 3} = \frac{T}{9}$

$A < \frac{4}{9} T$

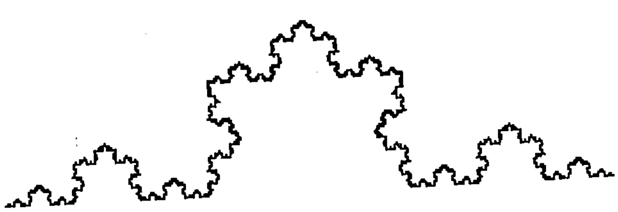
area $= \frac{T}{9 \cdot 9} = \frac{T}{81}$



$A < \frac{16}{81} T$

$= \left(\frac{4}{9}\right)^2 T$

Area of the Koch curve is $< \left(\frac{4}{9}\right)^n T$



$A = 0$

Box-counting dimension

$N(r)$ = minimum number of boxes of side length r needed to cover the shape.

Scaling hypothesis $N(r) = k\left(\frac{1}{r}\right)^d$

If this holds, d is the box-counting dimension.

Rules of logarithms

$$\mathrm{Log}(a^b) = b \cdot \mathrm{Log}(a)$$

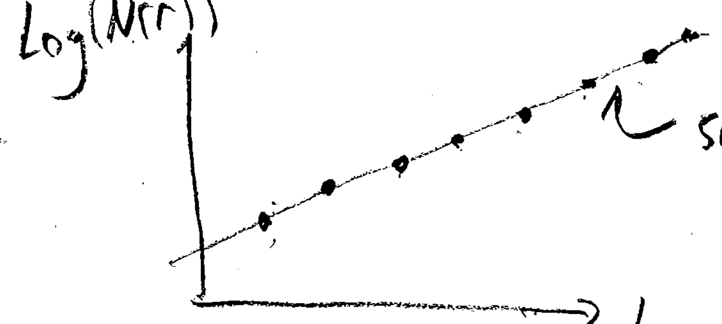
$$\mathrm{Log}(c \cdot d) = \mathrm{Log}(c) + \mathrm{Log}(d)$$

Take Log of both sides of the scaling hypothesis

$$\mathrm{Log}(N(r)) = \mathrm{Log}\left(k\left(\frac{1}{r}\right)^d\right)$$

$$= \mathrm{Log}(k) + \mathrm{Log}\left(\left(\frac{1}{r}\right)^d\right)$$

$$\underbrace{\mathrm{Log}(N(r))}_{y} = \underbrace{d}_{m} \cdot \underbrace{\mathrm{Log}\left(\frac{1}{r}\right)}_{x} + \underbrace{\mathrm{Log}(k)}_{b}$$



log-log approach

If we have a formula for $N(r)$, we can calculate d more directly

$$\text{Log}(N(r)) = d \cdot \text{Log}\left(\tfrac{1}{r}\right) + \text{Log}(k)$$

Solve for d

$$d \, \text{Log}\left(\tfrac{1}{r}\right) = \text{Log}(N(r)) - \text{Log}(k)$$

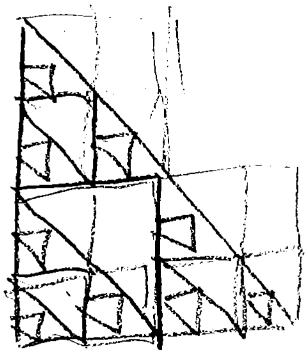
$$d = \frac{\text{Log}(N(r))}{\text{Log}(1/r)} - \frac{\text{Log}(k)}{\text{Log}\left(\tfrac{1}{r}\right)}$$

What happens to the right hand side as $r \to 0$? If $r \to 0$, $\frac{1}{r} \to \infty$

$\text{Log}(1/r) \to \infty$, and $\left.\frac{\overset{\text{constant}}{\text{Log}(k)}}{\text{Log}(1/r)}\right\} \to 0$

$$d = \lim_{r \to 0} \frac{\text{Log}(N(r))}{\text{Log}(1/r)}$$

limit approach



$N(1) = 1$

$N(\frac{1}{2}) = 3$

$N(\frac{1}{4}) = 9$

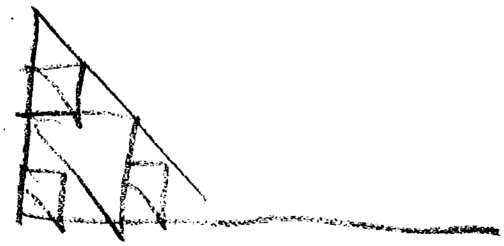
$N(\frac{1}{8}) = 27$

$N(\frac{1}{2^n}) = 3^n$

$$d = \lim_{\frac{1}{2^n} \to 0} \frac{Log(N(1/2^n))}{Log(1/(1/2^n))} = \lim_{n \to \infty} \frac{Log(3^n)}{Log(2^n)}$$

$$= \lim_{n \to \infty} \frac{\cancel{n} Log(3)}{\cancel{n} Log(2)}$$

$$= \frac{Log(3)}{Log(2)} \approx 1.58996$$



$$N\left(\frac{1}{2^n}\right) = 3^n + 2^n$$

$$d = \lim_{n\to\infty} \frac{\log(3^n+2^n)}{\log(2^n)}$$

factor out the larger

$$= \lim_{n\to\infty} \frac{\log\left(3^n\cdot\left(1+\frac{2^n}{3^n}\right)\right)}{\log(2^n)}$$

$$= \lim_{n\to\infty} \frac{\log(3^n)}{\log(2^n)} + \cancel{\frac{\log\left(1+\frac{2^n}{3^n}\right)}{\log(2^n)}} \quad \to 1 \to 0$$

$$= \lim_{n\to\infty} \frac{\cancel{n}\log(3)}{\cancel{n}\log(2)} = \frac{\log 3}{\log 2}$$