Sept 29 review 1

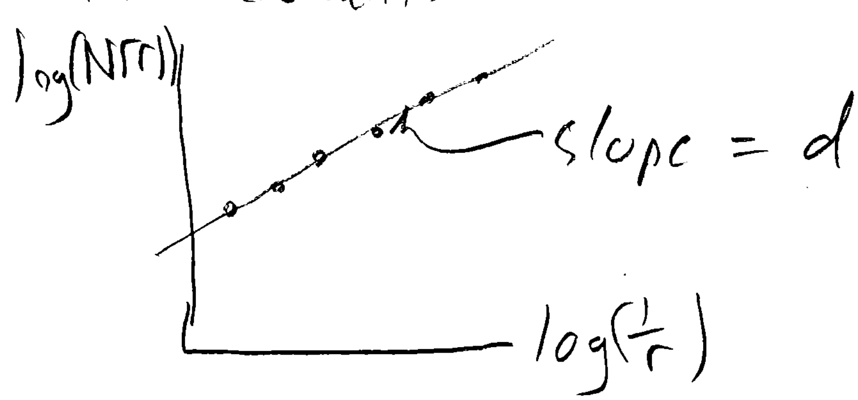
Scaling Hypothesis: the minimum number of boxes to cover a shape scales as

$$N(r) = k \cdot \left(\frac{1}{r}\right)^{d}$$

d — box-counting dimension

k — proportionality constant (depends on the shape)

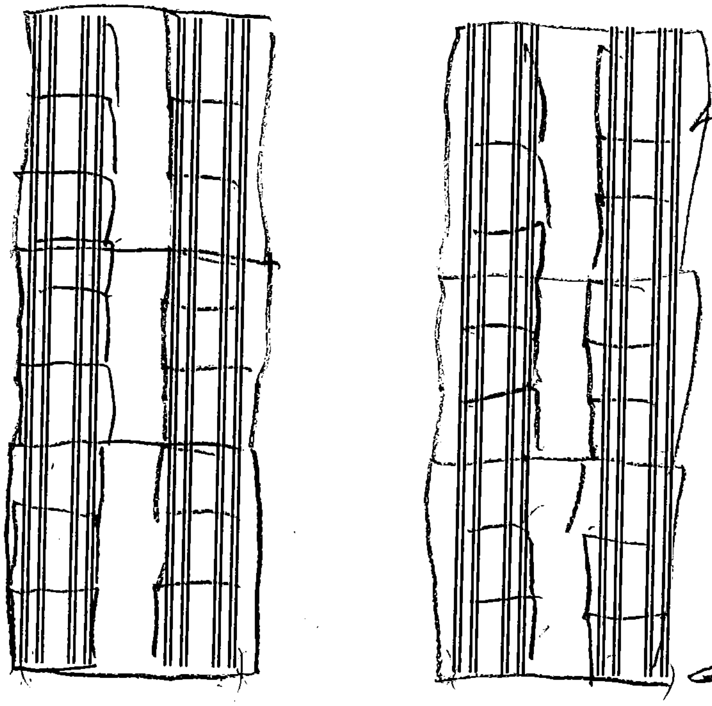
Measurements



(Experimental data)

If we have a formula for $N(r)$

then $$d = \lim_{r \to 0} \frac{\log(N(r))}{\log(1/r)}$$



line segment in the y-direction

$N(1/3) = 6$

$N(1/9) = 9 \cdot 4 = 6 \cdot 6$

$N(1/27) = 6^3$

Cantor Middle Thirds Set in the x-direction

Cantor set suggest using $r = 1/3, 1/9, 1/27, \ldots$

$$N(1/3^n) = 6^n$$

$$d = \lim_{1/3^n \to 0} \frac{\mathrm{Log}(N(1/3^n))}{\mathrm{Log}(1/(1/3^n))} = \lim_{n\to\infty} \frac{\mathrm{Log}(6^n)}{\mathrm{Log}(3^n)}$$

$$= \lim_{n\to\infty} \frac{\cancel{n}\,\mathrm{Log}(6)}{\cancel{n}\,\mathrm{Log}(3)} = \frac{\mathrm{Log}(6)}{\mathrm{Log}(3)} = \frac{\log(2\cdot 3)}{\mathrm{Log}(3)}$$

$$= \frac{\mathrm{Log}(2) + \mathrm{Log}(3)}{\mathrm{Log}(3)} = \underbrace{\frac{\mathrm{Log}(2)}{\mathrm{Log}(3)}}_{\text{dim(Cantor MTS)}} + \underbrace{\cancel{\frac{\mathrm{Log}(3)}{\mathrm{Log}(3)}}}_{\text{dim(line segment)}}^{\,1}$$