Reading

Read and make sure you understand FIS §1.1, §1.2, §1.3.

Problems

In writing down your solution you should justify every statement. If it is true, cite a Definition, Lemma, Proposition, Theorem, or Corollary etc. from the book or from class, or give a proof; if it’s false, provide a counterexample.

1. Let $F$ be a field. Prove the following statements:

   (a) The two neutral elements 0 and 1 in $F$ are unique.

   (b) Let $a \in F$ be an element. Then $-a \in F$ is unique. If $a \neq 0$ then $1/a \in F$ is unique as well.

   (c) $0 \cdot a = 0$ and $(-1) \cdot a = -a$ for every $a \in F$.

   (d) $-(a \cdot b) = (-a) \cdot b$ for every $a, b \in F$.

   (e) Let $a, b \in F$ be a pair of elements. Then the equation $a + x = b$ has a unique solution in $F$. If $a \neq 0$ then the equation $a \cdot x = b$ has a unique solution in $F$ as well.

2. Let $F$ be a field. Let $V$ be your favorite vector space discussed in class (e.g. $n$-tuples, matrices, polynomials) over the field $F$. Verify in detail that $V$ indeed satisfies all of the vector space axioms.

3. FIS §1.2 Exercises 1, 13, 21.