Yale University Department of Mathematics  
Math 225 Linear Algebra and Matrix Theory  
Problem Set 8  
Due in class Tuesday 14 November 2017

Reading

Read and make sure you understand FIS §4.1, §4.2.

Problems

1. Let $A$ and $B$ be two square matrices of size $n$. Prove or disprove the following statements
   
   (a) $\text{tr}(A + B) = \text{tr}(A) + \text{tr}(B)$,
   (b) $\det(A + B) = \det(A) + \det(B)$,
   (c) $\text{tr}(AB) = \text{tr}(A) \cdot \text{tr}(B)$,
   (d) $\det(AB) = \det(A) \det(B)$ (here you may assume that $n = 2$).
   (e) If $\det(A) = \det(B)$ then $\text{tr}(A) = \text{tr}(B)$.

2. Compute two determinants from FIS §4.2 Problems 5 – 12 and two determinants from FIS Problems 13 – 22.


4. Recall that a permutation matrix is a square matrix that has a single non-zero entry in each row and in each column, and these non-zero entries are equal to $+1$. Prove that the determinant of any permutation matrix is either $+1$ or $-1$.

5. Let $A$ be an $n \times n$ square matrix. Recall that $A$ is lower-triangular if $a_{i,j} = 0$ for every $1 \leq j < i \leq n$. $A$ is upper-triangular if $A^t$ is lower-triangular.
   
   (a) Prove that the determinant of any lower or upper triangular matrix is equal to the product of its diagonal entries.
   (b) Conclude that the determinant of a diagonal matrix is equal to the product of its diagonal entries.

6. Let $A$ and $B$ be two $n \times n$ matrices with $n \geq 3$. Assume that $A_{i,i} = B_{i,i}$ holds for every $1 \leq i \leq n$, where $A_{i,i}$ and $B_{i,i}$ are minors of $A$ and $B$. Prove that $A = B$. 