# 1 Preface

Subject of black hole quasinormal modes has been around for a while. Around 2002 it was suggested that the very highly damped modes might have some relevance for quantum gravity. I'll describe some computations in classical GR which were motivated by this conjecture. The results seem to contradict the simplest version of the conjecture, but still appear suggestive. I'll give a few speculations about what the result could mean.

# 2 Area quantization

The idea of area quantization is usually attributed to Bekenstein. In view of the formula

$$S = A/4G$$

it seems natural to suppose that the degrees of freedom responsible for the microstates of the black hole really live on the horizon. In fact, suppose the horizon is a "brick wall" built from elementary objects each of which has $k$ states. Then there will be a minimum amount of energy that the black hole can emit:

$$dU = TdS = T \log k$$

where $T = T_H$ is the Hawking temperature (and we set $k_B = 1$).

One therefore is tempted to hunt for a characteristic frequency of this form.

# 3 Quasinormal frequencies

The most obvious frequencies attached to a black hole are the *quasinormal modes*. These describe the "ringing" of the black hole when perturbed (at linear order), say by a small fluctuation of the metric, or a scalar field in the black hole background. The quasinormal frequencies $\omega$ are complex; real part gives oscillation while the imaginary part gives speed of exponential decay.

Numerical computations of the quasinormal frequencies for the Schwarzschild black hole in $d = 4$ yielded an interesting result: an infinite tower of quasinormal modes which asymptotically approach [Hod]

$$\omega = T_H \left( \log 3 + 2\pi i \left( n + \frac{1}{2} \right) \right).$$

This was regarded as possible vindication for the idea of black hole area quantization, with $k = 3$ — and even for its realization in loop quantum gravity, if you use $SO(3)$ [Dreyer].

This naturally leads to the questions: Is the numerical result indeed true analytically? Does it also hold for other types of black hole?

The first question was answered in the affirmative [Motl], using however a somewhat technical approach which looked difficult to understand and generalize. To answer the second question we introduced a new method for computing the asymptotic quasinormal frequencies.

## 4 The computation

We are studying the dynamics of some perturbation $\phi$ of the black hole — say a massless scalar field propagating in the black hole spacetime. To be concrete discuss Schwarzschild in $d = 4$. Separate variables in Laplace equation: writing

$$\phi(r, \theta, \varphi, t) = r\psi(r) Y_{lm}(\theta, \varphi) e^{i\omega t}$$

the radial $\psi(r)$ obeys a second-order differential equation in $r$. It has an irregular singularity at $r = \infty$, regular singularities at $r = r_H$ and $r = 0$. Introducing "tortoise coordinate" $x(r) = r + r_H \log(r - r_H)$, the equation can be written

$$\left[-\frac{\partial^2}{\partial x^2} + V[r(x)] - \omega^2\right]\psi(x) = 0$$

with the potential

$$V(r) = \left(1 - \frac{1}{r}\right)\left(\frac{l(l+1)}{r^2} + \frac{1}{r^3}\right).$$

Then define transmission and reflection coefficients $T(\omega)$, $R(\omega)$ as usual:

$$\psi \sim T(\omega)e^{+i\omega x} \text{ as } x \to -\infty, \quad \psi \sim e^{+i\omega x} + R(\omega)e^{-i\omega x} \text{ as } x \to +\infty$$

The quasinormal modes are the simultaneous poles of $T(\omega)$, $R(\omega)$. Roughly such a pole should correspond to a solution which is purely outgoing at both $x \to \pm\infty$. If there are no bound states (as for the black hole case) then these solutions can occur only for $\operatorname{Im}\omega > 0$. But they are rather subtle to find (or even define) since we are asking for the exponentially small solution to be absent. Can try to make sense of this by studying the power series expansion around $r = 1$: leads to an infinite continued fraction equation on $\omega$.

We take a different approach. Define the boundary condition at spatial infinity (irregular singularity) by analytic continuation to the line $\omega x \in \mathbb{R}$ where the two solutions $e^{\pm i\omega x}$ are oscillatory; then it's easy to say what it means for one to be absent. Define the boundary condition at the horizon (regular singularity) by the monodromy in the complex $r$-plane. Then in the large $\operatorname{Im}\omega$ limit we can study the equation by following a WKB contour from infinity to $r = 0$ and around the horizon. Requiring that this gives the expected monodromy leads to a constraint on $\omega$.

Note that the WKB contour has to pass through a turning point which (in large $\mathrm{Im}\,\omega$ limit) comes extremely close to the coordinate "singularity" at $r = 0$!

## 5 Results

For the Schwarzschild black hole in arbitrary dimension $d$, one finds

$$e^{\beta\omega} + 3 = 0$$

agreeing with the numerical prediction for $d = 4$.

For other black holes it's more complicated. For example, Reissner-Nordstrom in $d = 4$:

$$e^{\beta\omega} + 2 + 3e^{\beta_I\omega} = 0$$

Kerr in $d = 4$ [Keshet-Hod, Neitzke-Keshet]:

$$e^{\beta_1\omega+\mu_1} + 1 = 0$$

and a similar formula for total transmission modes,

$$e^{\beta_2\omega+\mu_2} + 1 = 0$$

where $\beta_{1,2}, \mu_{1,2}$ obey

$$\frac{1}{2}(\beta_1 + \beta_2) = \beta, \quad \frac{1}{2}(\beta_1\mu_1 + \beta_2\mu_2) = \beta\mu \quad (\mu = m\Omega)$$

and are given by certain elliptic integrals. (For example, $\beta_{1,2}$ are inverse transit times along certain complexified null geodesics, connecting the WKB turning points; similarly $\mu_{1,2}$ related to angular distance along these geodesics.)

All these results also confirmed numerically. (And similar formulas for other spins of perturbation.) Moreover the method has been extended to various other situations (dilaton gravity, AdS and dS black holes etc.)

The results seem to defy a simple interpretation in terms of area quantization: in particular they don't lead to any single characteristic frequency. However, they are still suggestively simple: so what do they mean?

## 6 Greybody factors

One possible interpretation emerges when we consider not just the quasinormal frequency but the full transmission amplitude $T(\omega)$ at large $\operatorname{Im}\omega$. (It is determined by a monodromy calculation similar to that described above.) This amplitude determines the "greybody factor" which filters the spectrum of Hawking radiation:

$$\sigma(\omega) \propto \frac{T(\omega)T(-\omega)}{e^{\beta\omega} - 1}$$

Historically this greybody factor has often been considered as an annoyance which is obscuring the perfect thermal spectrum. But it has also served as a clue to the quantum description of certain black holes!

For example, consider string theory on $T^4 \times S^1$. Get a 6-parameter family of solutions by studying objects with three charges $Q_1$, $Q_5$, $n$, mass $M$, and internal volumes $V$, $R$. Letting $g$ denote string coupling, consider the semiclassical limit $g \to 0$ with fixed $gQ_1$, $gQ_5$, $g^2 n$: if they are large then this is a black hole well described by classical action (Einstein-Maxwell plus dilaton), if small then well described by string perturbation theory. So try calculating the radiation from the black hole using string perturbation theory. (In a further "dilute gas" limit.) At low enough energies $\omega$ (and say for $\ell = 0$), this computation gives

$$\sigma(\omega) \propto \frac{1}{(e^{\beta_L \omega} - 1)(e^{\beta_R \omega} - 1)}$$

which doesn't look like the thermal spectrum; in fact it contains information about the underlying SCFT describing the degrees of freedom of the black hole ("effective string" with excitations traveling around to the left and the right, which have to interact with one another to create a quantum of Hawking radiation, with characteristic temperatures $\beta_L$, $\beta_R$.)

Now compare this to the Hawking computation, including the classically-computed greybody factor:

$$T(\omega)T(-\omega) \propto \frac{e^{\beta \omega} - 1}{(e^{\beta_L \omega} - 1)(e^{\beta_R \omega} - 1)}$$

We see that the spectrum of Hawking radiation computed using semi-classical spacetime action actually *agrees* with this SCFT spectrum!

So: reversing the historical order, you might say that the classical computation of the greybody factor gave a clue about an underlying quantum mechanical description of the black hole.

What would that description be? Our results for Schwarzschild and Reissner-Nordstrom make it look rather exotic, e.g. "tripled Fermi statistics", excitations involving both inner and outer horizons. For Kerr the situation seems a bit more conventional: two subsystems labeled by $\beta_{1,2}$, $\mu_{1,2}$ which interact with the environment only through processes with $dU_1 = dU_2$ and $dN_1 = dN_2$. Then the outside observer sees a system characterized by $\beta, \mu$. This answer is in some sense more stable against small perturbations.