M 365C

Fall 2013, Section 57465

Problem Set 1

Due Thu Sep 5

Problem sets will be assigned weekly, due on Thursday at the beginning of class, or in the mail slot outside my office door (RLM 9.134) no later than 10:00a Thursday. You are strongly encouraged to work together on the problems. However, you must write up your own solutions, independently.

Late homework cannot generally be accepted, because it creates extra work for the already-overworked TA.

The book has many more problems than I assign, and if you feel you want more practice, please take advantage of them. Come to office hours for additional problems if you need.

In your solutions to these exercises you may freely use any results proven in class or in Rudin chapter 1, without reproving them.

Exercise 1 (*Rudin 1.1*)

(For this exercise all you need to know about the real numbers $\mathbb{R}$ is that they form a field containing the field $\mathbb{Q}$ of rational numbers.)

Suppose $r \in \mathbb{Q}$, $r \neq 0$, and $x \in \mathbb{R}$, $x \notin \mathbb{Q}$. Prove that $r + x \notin \mathbb{Q}$ and $rx \notin \mathbb{Q}$.

Exercise 2 (*Rudin 1.3*)

Prove Proposition 1.15 in Rudin.

Exercise 3 (*Rudin 1.4*)

Let E be a nonempty subset of an ordered set S. Suppose α is a lower bound of E and β is an upper bound of E. Prove that $\alpha \leq \beta$.

Exercise 4

Suppose $a, b \in \mathbb{Q}$ with $a < b$. Let $E = \{x \in \mathbb{Q} \mid a < x,\ x < b\}$. Prove that $\sup E = b$. (Similarly $\inf E = a$, but you don't need to write out the proof of this part.)

Exercise 5 (*Rudin 1.5*)

Let $A \subset \mathbb{R}$ be nonempty and bounded below. Let $-A = \{-x \mid x \in A\}$. Prove that $\inf A = -\sup(-A)$.