M 382D: Differential Topology
Spring 2015

Exercise Set 5
Due: Wed Mar 4

Exercise 1. Guillemin/Pollack: Chapter 1, §5 (p. 32): 2 (just answers, needn't write out proofs), 5, 6, 10, 11

Exercise 2. Guillemin/Pollack: Chapter 1, §6 (p. 33): 10

Exercise 3. Define

$$M = \{[x, y, z] \in \mathbb{CP}^2 : x^2 + y^2 - z^2 = 0\} \subset \mathbb{CP}^2.$$

1. Prove that M is a 2-dimensional submanifold of $\mathbb{CP}^2$.

2. Consider the *pencil* (1-dimensional family) of *projective lines*

$$N_t = \{[x, y, z] \in \mathbb{CP}^2 : x + y + tz = 0\} \subset \mathbb{CP}^2.$$

   Here $t \in \mathbb{C}$. Define a projective line N_∞ which deserves to be called the limit of N_t as $t \to \infty$. Write an equation for N_∞.

3. For which t do M and N_t intersect transversely? For those t identify the manifold $M \cap N_t$.

4. Redo the problem with $\mathbb{RP}^2$ replacing $\mathbb{CP}^2$.

Exercise 4.

1. Let $f = f(x, y, z)$ and $g = g(x, y, z)$ be smooth functions defined on an open set $U \subset \mathbb{A}^3$, and suppose each has 0 as a regular value. Then $M = f^{-1}(0)$ and $N = g^{-1}(0)$ are submanifolds of $\mathbb{A}^3$ of dimension 2. Then M and N intersect transversely if and only if a certain condition on f and g holds. What is it?

2. Check your answer for the specific functions

$$f = x^2 + y^2 + z^2 - 1$$
$$g = (x - a)^2 + y^2 + z^2 - 1$$

   where a is a real parameter. For what values of a is the intersection transverse? Think about the geometric picture as well as the equations.